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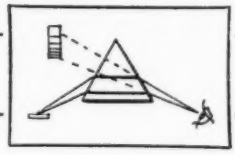

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Journal of the
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Sidelights on the Era of Young and Fresnel*

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(Received October 17, 1932)

I. THE WORK OF THOMAS YOUNG

THE proper way to write history, we are told, is to start with the present and work backwards into the past. Following this method, I preface this sketch of optics in the early nineteenth century—the age of Young and Fresnel—by quoting Jeans. At the New York dinner in his honor, in speaking of the new wave mechanics he said, parenthetically, "Of course they are not real waves, for there is no medium."

A much earlier sceptic is reported to have said, "If there were no deity it would be necessary to invent one." The originators of the undulatory theory had, however, no need to "invent" an ether; the science of the preceding centuries was saturated with ethers, electric, magnetic, gravitational, caloric.

Young[1] in his Bakerian lecture (1801) on the theory of light and colors states the four hypotheses upon which his undulatory theory of light was to be based, i.e.,

Hypothesis I

A luminiferous ether pervades the universe, rare and elastic in high degree.

Hypothesis II

Undulations are excited in this ether whenever a body becomes luminous.

Hypothesis III

The sensation of different colors depends on the different frequency of vibrations excited by light in the retina.

Hypothesis IV

All material bodies have an attraction for the etherial medium by means of which it is accumulated within their substance, and for a small distance around them in a state of greater density but not of greater elasticity.

In the background of these hypotheses lay the work of Huygens, of Hooke, and, as Young took pains to show by copious quotations, of Newton himself. In these passages selected from various portions of Newton's works, "the ether" is repeatedly mentioned, e.g.,

> It is to be supposed—that there is an etherial medium, much of the same constitution with air but far rarer, subtiler and more strongly elastic.
>
> And is not this medium the same with that medium by which light is reflected and refracted and by whose vibrations light communicates heat to bodies and is put into *fits of easy reflection and easy transition?*
>
> And doth it not readily pervade all bodies and is it not by its elastic force expanded through all

* Presented by invitation at the annual meeting of the Optical Society of America, October 13–15, 1932, at the Lake Placid Club, Lake Placid, N. Y. *Editor.*

[1] Works of Thomas Young, M.D., F. R. S., vol. I, pp. 140 *et seq.*

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> the heavens?—May not planets and comets and all gross bodies perform their motions in this etherial medium?

In discussing his third hypothesis Young cites Newton at some length; in effect,

> —that the parts of bodies, when briskly agitated, do excite vibrations in the ether which are propagated in every way from those bodies in straight lines and cause a sensation of light by beating and dusting against the bottom of the eye, something after the manner that vibrations in the air cause the sensation of sound by beating against the organs of hearing.[2]

And again,

> To explain colors I suppose that as bodies of various sizes, densities or sensations do by percussion or other action excite sounds of various tones and consequently vibrations in the air of different bigness, so the rays of light—excite vibrations in the ether of various bigness; the biggest, strongest or most potent rays the largest vibrations; and others shorter according to their bigness, strength or power, and therefore the ends of the capillamenta of the optic nerve—when the rays impinge upon them they must there excite these vibrations, which vibrations—will run along the aqueous pores or crystalline pith of the capillamenta, through the optic nerve into the sensorium and there, I suppose affect the sense with various colors according to their bigness and mixture; the biggest with the strongest colors, reds and yellows; the least with the weakest, blues and violets; the middle with green and a confusion of all with white—.

To these passages from Newton, of which only the most essential phrases are reproduced here, Young appends a brief *scholium* in which he asserts that since it is almost impossible to suppose each point of the retina to have an infinite number of particles "each capable of vibrating in perfect unison with every possible undulation, it becomes necessary to suppose the number limited, for instance to the three principal colors[3] *red*, *yellow* and *blue;* of which the undulations are related in magnitude nearly as the numbers 8, 7 and 6; and that each of the particles is capable of being put in motion more or less forcibly by unison; for instance, the undulations of green light being nearly in the ratio of $6\frac{1}{2}$ will affect equally the particles in unison with yellow and blue and produce the same effect as a light composed of those two species; and each sensitive filament of the nerve may consist of three positions, one for each color."

Thus in one comprehensive sentence was outlined the basis of the *Young-Helmholtz Theory of Color.*

The selection of yellow, instead of green, as one of primary colors[4] seems to have been dictated by a desire to maintain the supposed analogy between sound and light very much in vogue at the time. Young, like many of his contemporaries, was influenced by this analogy and by the conviction that undulations must be transmitted by longitudinal vibrations of the ether. Even after the existence of transverse waves in the transmission of sound had been experimentally demonstrated by Savart he continued to reject as unthinkable the existence of such motions in the ether.

Under these limitations the wave theory encountered new difficulties as the number of known phenomena increased.

Concerning the nature of heat and its relation to light he wrote (in his paper of 1801), "It was long an established opinion that heat consists in vibrations of particles of bodies and is capable of being transmitted by undulations through an apparent vacuum. This opinion has been of late very much abandoned. Count Rumford, Professor Pictet and Mr. Davy are almost the only authors who have appeared to favor it; but it seems to have been rejected without any good grounds and will probably very soon recover its popularity."

Two years later, having learned of the discovery by Ritter of chemical action beyond the violet he succeeded in producing diffraction of ultraviolet rays and thus estimating their frequency; and he asserted confidently, "If we had thermometers sufficiently delicate we might acquire by similar means (i.e., by producing diffraction) information still more interesting with respect to the rays of invisible heat discovered by Dr. Herschel.[5]

Thus, a half century ahead of his time Young had clearly in mind a spectrum extending in both

[2] Phil. Trans. I, 145 (1675).

[3] Reference 1, p. 147.

[4] Not later than 1817 green was substituted by Young for yellow and violet for blue (reference 1, p. 282).

[5] Reference 1, p. 191.

directions beyond the range of the eye, differing throughout only as to frequency and intensity.

That Young became acquainted with the discovery by Malus of polarization by reflection (1809) in the year 1810 appears from his review of the *Memoires d'Arcueil;*[6] but he does not seem to have grasped its bearing upon the undulatory theory at that time.[7]

Indeed as late as 1815 he wrote to Brewster, "With respect to my own fundamental hypotheses respecting the nature of light I become less and less fond of dwelling on them, as I learn more and more facts like those which Mr. Malus discovered."

Nor was he alone in having doubts. Arago, who had long since declared his zealous adherence to the undulatory theory, in opposition to La Place, Poisson, Biot and others, wrote to Young in 1818,[8] "There are still dispersion and polarization that demand a plausible explanation."

In the opening paragraph of his article on *Chromatics* written for the *Encyclopedia Britannica* in 1817 (sixteen years after launching his theory) he says, "But notwithstanding all that has hitherto been done it appears to be utterly impracticable, in the present state of our knowledge, to obtain a satisfactory explanation of all the phenomena of optics, considered as mechanical operations, upon any hypothesis respecting the nature of light that has hitherto been advanced."[9]

Later on, in the same article[10] he says, "Although the ingenuity of man has not yet been able to devise anything like a satisfactory reason for the reflection of a polarized ray in one case and its transition in another," etc., etc. "This difficulty," he declares in the closing paragraph of his paper, "will probably long remain, to mortify the vanity of an ambitious philosophy, completely unresolved by any theory."[11]

II. The Undulatory Theory in France

The undulatory theory, which had its origin in the wonderfully versatile mind of that English physician, jack of all trades and master of them all, Thomas Young, M.D.,[12] was in its completed form also the work of one man; a French engineer, Augustin Fresnel.

We may properly date the birth of the undulatory theory in France to the above-mentioned observation of Malus, who returned to France in 1801 after taking part in Napoleon's campaigns in Africa and the Near East. For some years he was employed in the building of fortifications in various parts of the country and later, in failing health, was given a position as examiner at the École Polytechnique in which he had been one of the earliest and most brilliant students.

In his spare time Malus studied optics and wrote a treatise on that science. It is not for any theoretical contribution, however, that he is remembered but for his discovery of polarization by reflection. Stories about the fortuitous observation upon which that famous discovery was based differ somewhat. According to Biot[13] the light of the westerly sun, reflected from one of the windows of the Luxembourg palace entered his dwelling in the neighboring Rue d'Enfer and was viewed through a doubly refracting crystal. Malus was amazed to see one of the double images of the sun disappear for certain positions of the crystal. The same evening he continued his observation, using a candle flame and a bowl of water and determined the angle of complete polarization for that liquid.

The immediate result of the research based upon this discovery was the election of Malus to the Institut de France but its full import was first made clear after his death in 1812, by Arago and Fresnel, who were also graduates of the École Polytechnique.

This celebrated school deserves mention here as the place of preparation of the three men memorable in the optical science of their day. It

[6] Reference 1, p. 249.
[7] Reference 1, p. 248.
[8] Reference 1, p. 382.
[9] Reference 1, p. 279.
[10] Reference 1, p. 382.
[11] Reference 1, p. 342.

[12] Young persistently disclaimed any other title. Like Helmholtz he seems to have entered the domains of sound and light from the physiological side; but the range of his interests was even broader than that of Helmholtz. He wrote on physics, astronomy, chemistry, botany, entomology, statistics, geodetics, egyptology, carpentry, tides, colloids, the structure of covered ways and bridges. His paper "On the Forms Best Suited for Teeth," refers not to dentistry but to cog wheels.

[13] Cajori, *History of Physics*, p. 115.

was founded in the year of "The Terror" (1794). On its staff from the beginning the foremost men of science and engineering in France were enrolled; La Place, La Grange, Poisson, Legendre, Biot, and later Arago, Malus, Ampere, Carnot, Fresnel. Arago in his autobiography tells[14] of the turbulence of the students in his day; of how, when in 1804 the moment came for the conversion of the French republic into an empire, many refused to take the prescribed oath of loyalty to the new regime.

The governor of the school reported the affair to the emperor:—"Monsieur Lacouee," cried Napoleon, who was surrounded by courtiers eager to applaud his every word and gesture, "you cannot retain in the school students who have shown such ardent republicanism. You must dismiss them." Then, "I want to see first of all their names and grades."

Upon seeing the list he got no farther than the first name which was that of the head of the class in the artillery.

"I will not drive out the best men in the school," said he. "Ah! if they were only at the foot!—Let it go at that M. Lacouee."

The great school has long survived its imperial patron and the stiff militarism prescribed by him; also, let us hope, the petty duties once demanded of its staff. Malus, and later Fresnel complained bitterly of the burden of correcting countless examination papers. The latter in a letter to Young (1824) writes,

"I am occupied from morning until night with my examinations at the École Polytechnique and shall not be freed for another fortnight from these painful labors which almost make me ill. It is a long time since I have been able to do anything new in optics."[15] (Freely translated.)

Arago had been assistant to Biot in triangulations, looking to the determination of the length of a quarter of the earth's meridian, in the course of which they had set up a station upon a mountain top on the island of Majorca. War broke out between France and Spain and they were interned in the citadel at Palma. Thence, with the friendly connivance of the commandant they had escaped in a fishing boat, had disappeared utterly and been given up as lost.

In the summer of 1809, however, they turned up at Marseilles and subsequently in Paris with tales of strange adventures—capture by Barbary pirates, escape, recapture; travels on foot disguised as moslems along the Algerian coast to Algiers, and residence for several months under the protection of the French legation in that city, pending an opportunity to return to France.[16]

These African exploits appealed strongly to his countrymen and Arago found himself a popular hero. A few days after his arrival in Paris he was nominated as academician and elected by a large majority (47 to 4) in spite of the opposition of La Place who urged him to withdraw in favor of Poisson. He was then twenty-three years old.

Verdet who half a century later compiled and edited the complete works of Fresnel insists that Arago was not elected member of the institute on the basis of his scientific achievements, which at that early day, were meager and unimportant, but solely because of his Algerian adventures.

This escape from slavery, and the consequent admission to the Academy, was one of a series of events entirely unconnected with each other which were to influence, one is almost inclined to say to make possible, the development of the undulatory theory in France.

Fresnel, then a young engineer employed under the government in the construction of roads and bridges, was a loyalist. He took part in a demonstration against the return of Napoleon from Elba (1815), was retired from his position and sent under surveillance to the neighborhood of Caen. Later, being in failing health, he was permitted to rejoin his mother's humble household at Mathieu.

Here in enforced leisure, without books or apparatus, he attacked the problem of the diffraction of light; found what he deemed a complete and satisfactory explanation based on Huyghen's theory of interference and an undulatory hypothesis quite of his own conception. Having confirmed his computations roughly but conclusively with a home-made micrometer, a lens of short focus, some supports constructed by the village carpenter and the usual hole in the

[14] Arago, *De ma Jeunesse.*

[15] Reference 1, p. 399.

[16] All these episodes are described in great detail in Arago's autobiographic sketch entitled *De ma Jeunesse.* [Arago, *Notices Biographiques.*]

shutter, he got in touch with Arago in Paris. Arago, who since his return from Africa had been studying polarization, was enthusiastic. He was converted to the views of Fresnel at once; urged the presentation of the results to the Academy; secured their acceptance by that body against La Place, Poisson, Biot and others—for at that time no one in France took Young's work seriously—pointed out to Fresnel that Young had been over much of the ground some years earlier with like results and should be granted priority. He put the substance of Young's papers into French for Fresnel who read no English and knew nothing of Young and his achievements. Fresnel wrote a courteous letter to Young in which he frankly acknowledged his chagrin at finding that he was himself not the first to propose and elaborate an undulatory theory of light. He also expressed his pleasure at the excellent agreement of their computations and the substantial identity of their conclusions. An extended and very friendly correspondence followed. Young at that time, 1815, was nearly at the end of contributions to optical theory, whereas the work of Fresnel was at its beginning.

Arago offered to collaborate with Fresnel and, best of all, persuaded the director general of roads and bridges, one of whose jobs it was to keep an eye on that dangerous young conspirator, to allow Fresnel to spend some months in Paris where there were books and instruments and the companionship of men of science.

Presently, i.e., in 1818, the Academy proposed as subject for the grand prize in the mathematical sciences for 1819 an exposition of the diffraction of light. According to Verdet,[17] the specifications for this prize essay were drawn up by influential members of the Academy such as La Place and Biot, not to further encourage Fresnel but in the assurance that a thorough-going discussion of diffraction would refute the fantastic dreams of Young and Fresnel and place the emission theory upon a firmer foundation.

Arago and Ampère, however, urged Fresnel to compete for the prize and the result was the memoir later crowned by the Academy and published in volume V of its Transactions.

[17] Verdet, *Introduction aux Oeuvres Complètes de A. Fresnel.*

The active collaboration of Arago and Fresnel was short lived. Arago, who had begun work on polarization as soon as he knew of the experiments of Malus, and had published an important paper in 1810, showing that polarized light was resolved by passing through thin plates of mica or selenite, etc., into two portions differently colored, joined with enthusiasm in the refinement of proofs of the undulatory theory and in the devising of new and ingenious experiments.[18] When it came to the theory of polarized light, however, the hypothesis of longitudinal waves proved inadequate. The difficulties which had driven Young to the verge of despair could not be solved.

The assumption of transverse vibrations led Fresnel to a beautifully simple and complete explanation, but Arago refused to accept such an absurd and unthinkable hypothesis, and Fresnel was left single handed to develop his theory of optics.

In rapidly failing health, with only such fragments of time as could be rescued from his work in the light-house service, to which he had been assigned by the government, and his onerous duties in the École Polytechnique he produced the system of lucid expositions which still bear his name. They commanded immediate and universal admiration for their clearness and elegance; led to membership in the Academy and shortly before his death from tuberculosis, in 1827, to the Rumford medal of the Royal Society of London.—But there were no notable conversions to the underlying principles either in France or England. Young's comment (in 1818) on the assumption of transverse vibrations was as follows:

"This hypothesis of Mr. Fresnel is at least very ingenious and may lead us to some satisfactory computations; but it is attended by one circumstance which is perfectly *appalling* in its consequences—it might be inferred that the luminiferous ether, pervading all space, and penetrating almost all substances, **is not only highly elastic but absolutely solid!!!"**

The old view was slow to die out. Cajori[19]

[18] Arago and Fresnel, *Memoir* presented to the Academy of Sciences in October, 1816, also *Annales de Chimie et de Physique*, 1819.

[19] Reference 13, p. 149.

places its demise as of May 6, 1850, on which day Foucault announced to the French academy that he had found the velocity of light in water less than in air. "From that moment," says Cajori, "Newton's emission theory was dead." Some adherents were unaware of its extinction even then.

The following talk with Brewster reported by Tyndall[20] probably took place several years later (1865?).

"In one of my latest conversations with Sir David Brewster," says Tyndall, "he said to me that his chief objection to the undulatory theory of light was that he could not think the Creator guilty of so clumsy a contrivance as the filling of space with ether in order to produce light."

[20] Tyndall; *Six Lectures on Light*, p. 49.

Some of you may recall reading the crusty retort of the militant agnostic (Tyndall) to the effect that[20] "the quarrel of science with Sir David, on this point, as with many estimable persons on other points, is that they profess to know too much about the mind of the Creator."

As for me, I might easily have heard Tyndall make that rather ill-natured remark had I chanced to be in New York that wintry evening in 1872 instead of being a sophomore in Ithaca.

That was the parting shot. The old guard had passed on; a new generation had gradually taken their place. The Young-Fresnel theory, or as they still call it in France, the Huygens-Fresnel theory was supreme and so remained until the time came for it to be translated into the broader terms of the electromagnetic theory of light.

Sensibility to Color-Temperature Change as a Function of Temperature*

DEANE B. JUDD, *Bureau of Standards, Washington, D. C.*

(Received October 6, 1932)

Determinations at the same brightness by six normal observers have been made of the color-temperature difference corresponding to a chromaticity difference just doubtfully perceptible over the range of 1800 to 11,000°K. It has been found that this temperature difference corresponds closely to a constant difference in the spectral centroid of light (Priest's empirical relation); it is also closely proportional to the square of the color temperature (Davis' representation of Priest's empirical relation); furthermore it corresponds with good approximation to a constant difference in the "red" trilinear coordinate (Judd's empirical relation). These experimental results have therefore approximately checked three empirical relations previously derived from less complete data. Priest's spectral-centroid relation and Davis' representation of it both agree so closely with the experimental data that the discrepancies are not known to be real, but Judd's empirical relation yields discrepancies about twice as large as the experimental uncertainty. The verification of Priest's spectral-centroid relation confirms the tentative conclusion by Davis and Gibson that between 2000 and 3000°K a given color-temperature difference causes for the normal observer nearly the same size chromaticity difference regardless of whether the radiators be viewed directly or through a blue filter not more highly selective than a "daylight filter." This result has a bearing on the phenomenon of "color constancy"; that is, it helps to explain why the colors of objects are approximately constant for illuminants differing as widely as incandescent-lamplight and natural daylight.

I. INTRODUCTION

WHEN color temperature is used, it is usually important to know the smallest interval in temperature which can be detected under the given experimental conditions. In particular, when color temperature is used for the specification of light sources in colorimetry, it is important to know this temperature interval because the relative distinctions between the various sources may then be determined and tolerances established.

Previous experimental data on this question are somewhat meager. They may be represented as one or another of three empirical relations; first, that the temperature interval corresponding to a given chromaticity difference is about the same as that corresponding to a constant change in the spectral centroid of light; second, that this interval varies approximately as the square of the color temperature; and, third, that it is about the same as that corresponding to a constant change in the r-coordinate of the color represented on a certain Maxwell color-triangle.

The first relation was discovered empirically by Priest[1] from experimental data on the precision of color-temperature settings; he states that his data showed no certain deviation from this relation, but that in individual cases the precision might vary by a factor approaching 2. It is to be noted that the range of color temperature covered by Priest's data is 3000 to 20,000°K, which fails to include most of the usual temperatures produced by incandescent lamps (2000 to 3000°K). The second relation was proposed by Davis[2] as a simpler statement of the spectral-centroid relation; he discovered empirically that the differences between these two relations are scarcely significant. The temperature interval given by this relation is also expressible as that

* Publication approved by the Acting Director of the Bureau of Standards of the U. S. Department of Commerce.

[1] I. G. Priest, *The Colorimetry and Photometry of Daylight and Incandescent Illuminants by the Method of Rotatory Dispersion*, J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 1190–1191 (1923).

[2] R. Davis, *A Correlated Color Temperature for Illuminants*, Bureau Standards J. Research **7**, 659–681 (1931). This simpler statement of the spectral-centroid relation might have been deduced by combining two previous findings, one by Gibson (see footnote 10, p. 12) concerning a spectral-centroid relation between incident and transmitted light for daylight filters, the other by Langmuir and Orange (Trans. A.I.E.E., **32**, 1944–1946 (1913)) concerning a similar relation involving reciprocal temperature. The mathematical analysis on which this latter finding is based was given later by Foote, Mohler and Fairchild, J. Wash. Acad. Sci. **7**, 545–549 (1917), and Gage, Trans. I.E.S. **16**, 428–429 (1921) also called attention to this relation.

corresponding to a constant change in reciprocal temperature.

The third relation is a special case of a more general relation discovered empirically by Judd[3] through an examination of all data available on chromaticity sensibility. Data referring to change in color temperature constitute a relatively small part of this whole. The general relation has been found to be in error in some cases by as much as 50 percent. There is no reason to suspect that this special case is any more reliable except that it agrees rather closely with the first two empirical relations which were obtained independent of it.

It is the purpose of the present paper, first, to give an account of a more adequate experimental determination of chromaticity sensibility as a function of temperature, and second, to compare the new experimental results with the three relations enumerated.

II. Apparatus and Procedure

The apparatus used consisted of a color comparator[4] for incandescent lamps combined with a Martens photometer,[5] a Davis-Gibson 2450-to-6500°K filter[6] and suitable screens made up of alternate opaque and transparent lines (half-tone screens) for reducing brightness without introducing any important change in chromaticity. By varying the current through the 400-watt lamps used in the color comparator, color temperatures of about 1800, 2200 and 2900°K were obtained, and the addition of the liquid filter yielded the two other color temperatures investigated: about 5000 and 11,000°K.

The matt surfaces of the color comparator, each surface illuminated by a 400-watt lamp, were viewed through the Martens photometer which, by virtue of a special diaphragm, provided a 2° circular observing field divided along its vertical diameter and surrounded by a dark field. The room containing the apparatus was dimly lighted. By the use of the three half-tone screens and by varying the distances of the lamps from the matt surfaces it was possible to maintain the retinal illumination nearly constant at 30 photons for three of the values of color temperature at which chromaticity sensibility was determined: 2200, 2900 and 11,000°K. At 1800 and 5000°K a converging lens placed in front of each lamp raised the illumination to the same value.

The color temperature of one-half of the field was set by an operator and kept constant by him throughout each group of ten settings; the operator also made adjustments of the half-tone screens and the lamp distances so that the retinal illumination corresponding to the observing field was about 30 photons regardless of the color temperature. The observer controlled the voltage applied to the lamp illuminating the other half of the observing field by operating rheostats with one hand, and, with the other hand, rotated the circle of the Martens photometer in order to eliminate such brightness differences as occurred from varying the voltage of the lamp incident to the settings of color temperature.

Each of six observers made 300 settings, 100 settings on each of three separate days. The general procedure was to make ten "greatest-imperceptible-difference" (GID)[4] settings for one color temperature, then ten more GID settings at another color temperature, and so on for all five color temperatures. Then, after five minutes rest, the observer made ten "least-perceptible-difference" (LPD)[4] settings at each of the five color temperatures in the same order as before. The order of color temperatures for the first observing day was 1800, 2200, 2900, 5000 and 11,000°K; for the second observing day it was 11,000, 5000, 2900, 2200 and 1800°K; and for the final day it was 2900, 2200, 5000, 1800 and 11,000°K.[7] After each setting the operator recorded the voltage of the lamp controlled by the observer.

[3] D. B. Judd, *Chromaticity Sensibility to Stimulus Differences*, J. Opt. Soc. Am. **22**, 72–108 (1932).

[4] D. B. Judd, *Precision of Color Temperature Measurements under Various Observing Conditions; A New Color Comparator for Incandescent Lamps*, Bureau Standards J. Research **5**, 1161–1177 (1930). (In Fig. 2, p. 1169 of this paper, a decimal point should be placed before each of the numbers on the ordinate scale.)

[5] F. F. Martens, *Über ein neues Polarisationsphotometer*, Phys. Zeits. **1**, 299–303 (1900).

[6] R. Davis and K. S. Gibson, *Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature*, Misc. Pub., Bur. Stds., No. 114, 68–69; January 21 (1931).

[7] Later analysis showed that order of presentation of stimuli did not affect sensibility importantly.

III. Results and Discussion

The reduction of these data follows the general plan outlined in a previous paper[4] except that we are not here concerned with median settings. The average between the greatest imperceptible difference and the least perceptible difference, (GID+LPD)/2, is computed for each of the five groups of 20 settings made by each observer on each day at each of the five color temperatures; also the probable error of a single observation, PE_{single}, is computed from each group of ten settings. As before it is found that on the average:

$$3.3\ PE_{single}=0.5(GID+LPD).$$

We take, therefore, either 3.3 PE_{single} or 0.5(GID+LPD) interchangeably as a measure of the "just doubtful difference" which is the discriminatory unit in terms of which we have chosen to express the experimental results. The final values of the doubtful difference (Table II) are computed as $[3.3\ PE_{single}+0.5(GID+LPD)]/2$. The doubtful difference is a difference sufficiently large to be detected by the observer in about 92 out of each 100 attempts. Note that we deal in this paper only with relative values of the doubtful difference as a function of temperature. For absolute values of the doubtful difference for various observing conditions, reference must be had to a previous paper.[4]

1. Reduction to average doubtful difference

In presenting the average results, it is easier to appreciate the uncertainty arising from individual variations if a reduction is first made to the average size of the doubtful difference; that is, we plan to eliminate that part of the individual variation which can be ascribed either to abnormally high sensibility throughout the whole range of color temperature or to abnormally low sensibility. As a first step in this reduction we compute at each color temperature the ratio for each observer of the doubtful difference to the mean doubtful difference for all observers; the average of these ratios over the five color temperatures is given in Table I. It is to be noted from the second column, headed 0.5(GID+LPD), that some observers set differences as just doubtful nearly twice as large as some others; also from the third column (PE_{single}) that some observers make settings of only about half the precision of some others; and it might therefore be said in a sense that sensibility to color-temperature change varies by nearly a factor of two from individual to individual. It is not to be supposed, however, that a real superiority in vision for some observers compared to certain others has been established. It is quite probable that the manner of manipulating the rheostats and the brightness adjustment of the Martens photometer in conjunction with eye movements is the basis for all the individual variation shown. For example, the arrangement of the observers according to the precision of setting (Table I, third column) corresponds closely to the amount of practise they have had at making chromaticity matches; that is, the smallest probable errors have been obtained by the most experienced observers. This has resulted in spite of the fact that no emphasis was placed on making precise settings; the emphasis was rather on the adoption of a definite technique of manipulating the controls and the maintenance of this technique throughout an entire run of 100 settings.

TABLE I. *Individual variations of the doubtful difference.*

Observer	Average ratio of the doubtful difference for each observer to the mean doubtful difference 0.5(GID+LPD)	3.3 PE_{single}
DBJ	1.07	0.85
RSH	1.33	1.26
GKW	1.09	1.05
MEB	0.97	1.22
IGP	0.78	0.80
KSG	0.76	0.82

2. Individual variations

Fig. 1 shows by the plotted points the individual results reduced to the average doubtful difference for the six observers. This reduction was made simply by dividing each value of doubtful difference obtained from a given observer by the corresponding average ratio given in Table I, and then averaging the resulting values of 0.5(GID+LPD) and 3.3 PE_{single}. The doubtful differences are shown in terms of three parameters, spectral centroid of light, λ_c, reciprocal color temperature, $1/\theta$, and red coordinate, r; hence any one of the three plots serves to indicate the individual variation. It is to be

noted that the points are scattered for each range of color temperature not only with respect to the ordinate (size of the doubtful difference) but also with respect to the abscissa, reciprocal temperature. The first sort of scatter suggests, of course, that some observers respond best to differences in color temperature at one part of the temperature scale, others at another. The amount of this scatter is in each case nearly one-third the mean ordinate. However, this scatter probably refers to experimental error rather than to significant individual variations in characteristics of vision.[8] It may not be clear why there should be any scatter with respect to reciprocal color temperature at all because the lamp illuminating the left half of the field was kept at the same color temperature for all observers. Strictly speaking, however, the abscissa refers to the reciprocal of the color temperature of the right half of the field, that is, to the reciprocal of the color temperature over which the observer had control. This color temperature was by no means constant; it varied from observer to observer in some cases by nearly two doubtful differences. Some of this variation, perhaps about one-third of it, may be ascribed to the uncertainties introduced by the chromatic aberration of the lenses used for two of the color temperatures, to non-uniformities in the half-tone screens, to experimental error, and to the slight aging of the lamps during their 50 hours of use. The major part of these differences are true individual differences, however, and probably refer to difference in yellow-blue sensitivity across the vertical median line of the retina. It has already been shown that such a difference exists for a single observer depending on the field size used;[4] so it is not surprising that different observers with the same field size should show it. These results emphasize the danger of departing from the strict substitution method when light sources are being compared as to color temperature.

3. Comparison with the empirical relations

The three empirical relations mentioned in the introduction may be indicated symbolically:

$$\Delta\lambda_c = K_1, \tag{1}$$

$$\Delta(1/\theta) = K_2, \tag{2}$$

$$\Delta r = K_3, \tag{3}$$

where the symbol, Δ, stands for "doubtful difference," and $\Delta\lambda_c$ is to be read "doubtful difference in spectral centroid of light," $\Delta(1/\theta)$, "doubtful difference in reciprocal color temperature," and so forth; K_1, K_2 and K_3 are constants, independent of color temperature, which characterize the observer and the experimental conditions. These three empirical relations are represented by the three horizontal lines appearing in Fig. 1; the degree of agreement of the experimental results with these empirical relations can be seen from the plotted points which represent individual results and from the broken lines which connect points representing for each color temperature the average for all observers.

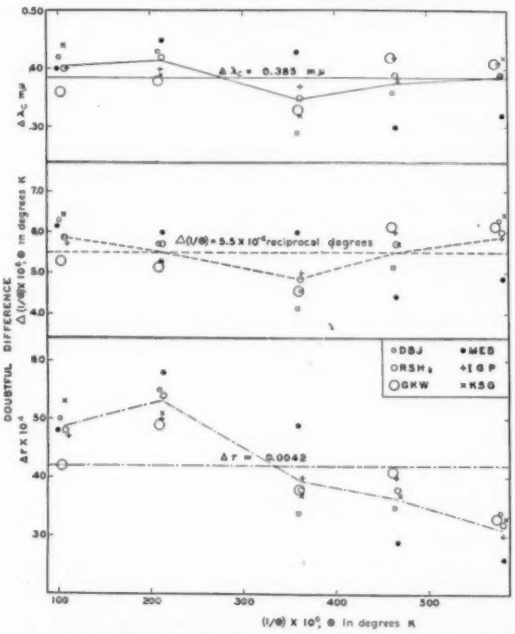

FIG. 1. Individual experimental values of the doubtful difference expressed according to the three parameters suggested by the three empirical relations. If one of the empirical relations represented the experimental results perfectly the corresponding plotted points would not deviate significantly from a horizontal line. Approach to the horizontal indicates, therefore, the degree of approximation. The scales are adjusted so as to afford a fair comparison of the three relations.

It will be seen that all three relations represent fairly well the experimental results. However, the

[8] Analysis shows that variation from day to day accounts for a large part of this scatter.

agreement is particularly good in the case of the spectral-centroid relation discovered by Priest, and in the case of its restatement in terms of color temperature squared (difference of reciprocal color temperature) by Davis. Either of these two relations represents the experimental results so closely that no attempt has been made to find a better law.

In order to show what variation of the doubtful difference in color temperature ($\Delta\theta$) as a function of color temperature is called for by these three empirical relations, it is convenient to write them in other terms:

$$d\theta/dE = -K_1 d\theta/d\lambda_c, \tag{1a}$$

$$d\theta/dE = -K_2 d\theta/d(1/\theta) = K_2\theta^2, \tag{2a}$$

$$d\theta/dE = -K_3 d\theta/dr, \tag{3a}$$

where dE is the chromaticity difference,[9] expressed as the number of doubtful differences, corresponding to the difference in color temperature, $d\theta$. The ratio, $d\theta/dE$, may be read "doubtful difference in color temperature" which is also represented by $\Delta\theta$. If the constants, K_1, K_2 and K_3 be solved for from relations (1a), (2a) and (3a), comparison with relations (1), (2) and (3) indicates that $d\lambda_c/dE$, $d(1/\theta)/dE$, and dr/dE are the same (except for algebraic sign) as $\Delta\lambda_c$, $\Delta(1/\theta)$, and Δr, respectively. The change in sign arises from the fact that λ_c, $1/\theta$, and r are all decreasing functions of θ.

Fig. 2 shows the right-hand members of relations (1a), (2a) and (3a) plotted as ordinate against color temperature, θ, as abscissa. The plotted points (circles) refer to the average experimental values of $d\theta/dE$. Except for the highest temperatures, these curves and points are shown twice, the scale for the upper set being ten times that for the lower. It is again demonstrated that relations (1) and (2) are in good agreement with experiment.

Table II shows the degree of agreement of these three empirical relations with the experimental results more specifically than can be seen from Figs. 1 and 2. It is seen that the average difference expressed in percent between the spectral-centroid relation and the experimental result is 4.9. Relation (2) departs from the experimental results by scarcely more, 5.1 per-

[9] E for "*Empfindung*," a fairly common usage among European writers; see also footnote 3, p. 8, for reference to a paper in which this symbol is used extensively.

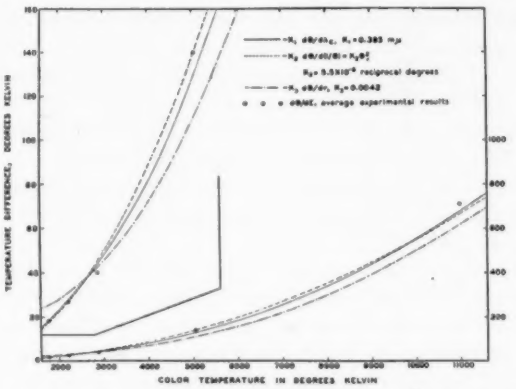

FIG. 2. Mean experimental values of the doubtful difference in color temperature as a function of color temperature compared to the three empirical relations. The rapid variation of the doubtful difference expressed in terms of color temperature compared to its slow variation in the other parameters (see Fig. 1) indicates the relative inconvenience of color temperature for the specification of illuminants. Reciprocal color temperature is suggested as a substitute parameter.

TABLE II. *Check of the three empirical relations against average experimental results.*

Empirical relation	θ	From the experimental results $-d\lambda_c/dE = K_1 = \Delta\lambda_c$	Deviation from mean	Average deviation, percent of mean
$\frac{d\theta}{dE} = -K_1\frac{d\theta}{d\lambda_c}$	1,764	0.386	0.001	
	2,213	.375	.010	
	2,875	.348	.037	
or $\Delta\lambda_c = K_1$	5,046	.415	.030	
(Priest)	10,990	.404	.019	
	Mean	0.385	0.019	4.9
		$(d\theta/dE)(1/\theta^2) = K_2 = \Delta(1/\theta)$		
$\frac{d\theta}{dE} = +K_2\theta^2$	1,764	5.87×10^{-6}	0.34×10^{-6}	
	2,213	5.50	.03	
	2,875	4.86	.67	
or $\Delta(1/\theta) = K_2$	5,046	5.52	.01	
(Davis)	10,990	5.89	.36	
	Mean	5.53	0.28	5.1
		$-dr/dE = K_3 = \Delta r$		
$\frac{d\theta}{dE} = -K_3\frac{d\theta}{dr}$	1,764	0.00310	0.00107	
	2,213	.00365	.00052	
	2,875	.00395	.00022	
or $\Delta r = K_3$	5,046	.00531	.00114	
(Judd)	10,990	.00486	.00069	
	Mean	0.00417	0.00073	17.5

cent. Relation (3) differs from the experimental results by an average of 17.5 percent.

Probably these comparisons are sufficient to justify the conclusion that relation (3) fails to represent normal sensibility exactly; that is, the discrepancies are about twice as large as the overall experimental uncertainty; but the comparisons are insufficient to establish that either of the other two relations is in error. Although relation (1) agrees slightly better with the experimental mean doubtful difference than does relation (2), this slightly increased agreement is, of course, not significant. Hence, in the remainder of the paper we take relation (2) to represent the doubtful difference in color temperature because of its greater simplicity; that is, we take:

$$d\theta/dE = -5.5 d\theta/d(1/\theta) \times 10^{-6} = 5.5\theta^2 \times 10^{-6} \quad (2b)$$

in degrees Kelvin per doubtful difference for observation by one eye of a 2° circular field yielding a retinal illumination of 30 photons with dark surroundings. With larger fields, binocular vision, and higher illuminations the doubtful difference may be reduced to one-fifth of these values.

Relation (2b), which has been chosen because of its simplicity, has, it appears, no fundamental significance; it approximates the true relation between the color-temperature limits 1700 and 11,000°K. Relation (3), which is a slightly poorer approximation between these limits, touches more closely on the actual phenomena; for example it indicates correctly the rise to infinity of $d\theta/dE$ as θ approaches zero. This corresponds to the fact that the stimuli defined by Planck's law for very low temperatures are visually identical (saturated red).

The form of relation (2) suggests that reciprocal color temperature would be a more convenient parameter for general use than color temperature, itself, because differences in reciprocal color temperature are proportional to the corresponding chromaticity differences. The use of reciprocal temperature instead of temperature is already fairly prevalent because of the form of the Wien radiation law.

4. Daylight filters and color constancy

From the verification of relation (1) there follows immediately an important relation having to do with blue filters of such transmissive characteristics that they serve to convert radiant energy of low color temperature (2000 to 3000°K) to high color temperatures (3000 to 12,000°K). Certain of these filters are known as "daylight filters," because of their use in producing artificial daylight in its various phases. It was pointed out in 1925 by Gibson[10] that for a given artificial daylight filter a linear relation exists between the spectral centroid of light at various color temperatures and that of the same light transmitted by the filter. He gave a graph demonstrating for seven artificial daylight filters of considerably varying transmissive characteristics not only that this relation is linear but also that the slope of the straight line is nearly unity. From the linearity of the relation we may conclude that a change in the color temperature of an incandescent source corresponding to a given chromaticity interval will produce a change in chromaticity of the light transmitted through such a filter equal to a constant times the given chromaticity interval regardless of the color temperature of the source. Furthermore, from the fact that the slope of the straight line is nearly unity, we may conclude that the chromaticity interval is nearly the same for any color temperature of source regardless of whether the two lamps exhibiting the color-temperature interval be viewed with the filter or without it, provided, of course, that the brightness is still sufficient for daylight vision. This conclusion, contingent on the validity of Priest's spectral-centroid relation, has already been used by Davis and Gibson[11] when they pointed out that the use of their filters in the determination of color temperature would probably yield just as high a precision as could be obtained without them.

As a demonstration of the accuracy of this conclusion there is compared in Fig. 3 the doubtful difference according to the color-temperature-squared relation (2a) with the doubtful difference computed from the assump-

[10] K. S. Gibson, *Spectral Centroid Relations for Artificial Daylight Filters*, J. Opt. Soc. Am. and Rev. Sci. Inst., 11, 473–478 (1925).

[11] R. Davis and K. S. Gibson, *Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature*, Misc. Pub. Bur. Stds., No. 114, 79, January, (1931).

tion that the interposition of an artificial daylight filter before two lamps makes no change in the size of the chromaticity difference between them. This latter doubtful difference is found by multiplying a doubtful difference in color temperature of source, $5.5\theta_s^2 \times 10^{-6}$, by the ratio of the change in color temperature of the transmitted light to the change in color temperature of the incident light, $d\theta/d\theta_s$, θ_s being the color temperature of the incident light from the source.

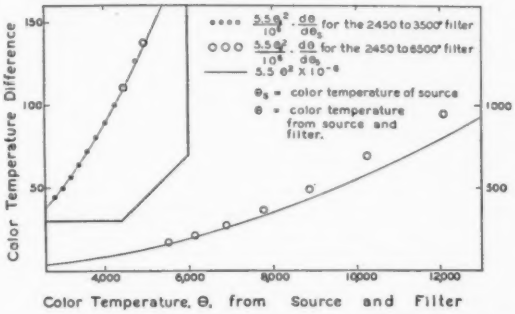

FIG. 3. Comparison of the doubtful difference in color temperature according to relation (2a) with that obtained by the assumption that the interposition of a daylight filter leaves the size of the chromaticity difference between two light sources unchanged. It is seen that this assumption is justified for the Davis-Gibson 2450-to-3500° filter, but not quite for the 2450-to-6500° filter which increases the chromaticity difference slightly.

Fig. 3 shows this comparison for two Davis-Gibson filters, 2450-to-3500°K and 2450-to-6500°K.[12] The curve shows the true variation of the doubtful difference (relation (2a)); the small circles show the variation of the doubtful difference computed from the assumption that the interposition of the 2450-to-3500°K filter leaves the size of the chromaticity difference unchanged; the large circles refer to the 2450-to-6500°K filter.

It is seen from Fig. 3 that the interposition of the less dense blue filter has made no definite change in the size of the chromaticity difference; that is, observation through this filter of two lamps just doubtfully differing in color temperature would result in the lamps still being judged just doubtfully different. The more dense filter slightly magnifies the chromaticity difference although the increase is scarcely significant. This is in conformity with the previous tentative conclusion by Davis and Gibson. We conclude, then, for all artificial daylight filters, and probably for most filters of whatever type that are not more highly selective than these, that the size of the chromaticity interval remains substantially unchanged regardless of whether the two nearly identical stimuli affect the eye immediately or after having passed through the filter.

It should be noted that this conclusion bears directly on a long known phenomenon called color constancy.[13] Color constancy refers to the fact that the colors of the usual objects of vision, which depend largely on selective reflection of light by surfaces, are nearly constant independent of the illuminant throughout a wide range. The classic illustration of color constancy is the fact that "white" objects such as paper, bleached linen and fresh snow are perceived as "white" and universally described as "white" even though illuminated by sources of widely varying spectral distribution of energy. Thus (says Kroh, p. 181), "we name and see the leaves of a book, in which we are reading, white, in spite of the fact that the reflected light may be the reddish-yellow light of a candle, the yellowish-green summer light in the woods, or the bluish-green light of a veranda having a glass roof of that color."

Long discussion led many to the conclusion that color constancy is a question of perception. It is said that a "white" object is seen as a "white" object under various illuminants of widely different spectral distribution of energy largely because the observer recognizes it and knows it to be "white." Although retinal adaptation tends in the same direction, an essential factor in color constancy is said to be a "painting

[12] See footnote 6, p. 8. This paper gives data for finding $d\theta/d\theta_s$.

[13] E. Hering, Grundzüge der Lehre vom Lichtsinn, aus Graefe-Sämisch Handb. d. ges. Augenheilk., XII, 1905 and 1907. H. v. Helmholtz, Handb. d. Physiol. Optik, 3d Ed., **2**, 233, 243 (1911). D. Katz, *Die Erscheinungsweisen der Farben und ihre Beeinflussung durch die individuelle Erfahrung, Ergänzungsband VII*, Zeits. f. Psychol., 1911. E. R. Jaensch, *Über Farbenkontrast und die sog.* Berücksichtigung farbigen Beleuchtung, Zeits. f. Sinnesphysiol. **52**, 165–180 (1921). O. Kroh, *Über Farbenkonstanz und Farbentransformation*, Zeits. f. Sinnesphysiol. **52**, 181–216, 235–273 (1921). A. Gelb, *Die "Farbenkonstanz" der Sehdinge*, in Handb. d. normalen u. pathologischen Physiol. 12, 1st half, Receptionsorgane II, 594–678; Berlin, Springer; 1929.

over of things with memory color" which makes them appear as they were previously experienced in daylight.

Although the present results do not deal directly with the color of objects seen under different illuminants, they may be stated (as is seen presently) as a case in which a change in illuminant has produced little or no change in the size of the color intervals between objects. Let us see what connection this constancy of color intervals may have with color constancy.

The phenomenon of color constancy is commonly described as if colors might be spoken of in an absolute sense, but in the usual descriptions of color constancy (such as that of Kroh, cited above) it is evident that other objects than that expressly referred to are present in the field of view; that is, we are not concerned with the constancy of the color of a uniform object filling the whole field of view, but rather with the color of an object seen against a background. If we had some pale yellow objects lying on a white background, and changes in the spectral distribution of the illuminant resulted in such objects appearing indistinguishable from the background (or even if the distinction became notably less), we would not think of speaking of "color constancy" in such circumstances. Hence, some degree of constancy of the color intervals must accompany every case of "color constancy"; and if it can be shown that the color intervals between objects are constant independent of illuminant because of purely retinal activity, we shall have gone a long way toward establishing the retina as the seat of the processes essential to color constancy.

We may construct from the results presented in Fig. 3 a quantitative example of the constancy of color intervals between the members of a series of yellow samples of a certain type of spectral reflectance. The series of black-body colors from 2000 to 3000°K may be thought of as produced by a series of objects varying from yellow to white illuminated by incandescent-lamplight at a color temperature of 3000°K. Viewing these lights through a daylight filter is, of course, equivalent to illuminating the hypothetical samples with daylight. The fact that the chromaticity distinction between two lamps is very little changed by observation through a daylight filter means for this case that the chromaticity distinction between a pale yellow sample and a white sample will be very little changed by a shift in illuminant from incandescent-lamplight to daylight. Furthermore, it is evident from the manner in which these data were obtained that no question of a possible effect of memory color or recognition of objects can arise; the conclusion holds, therefore, independent of perception or memory color.

This special case strongly suggests that all cases of color constancy involve a high degree of constancy of the size of the color intervals between objects entirely independent of perception or memory color. Another special case of this tentative generality has already been pointed out by A. Kohlrausch.[14] He links the approximate constancy of achromatic colors to the approximate validity of Weber's law concerning sensibility to brightness change. If the tentative conclusion, supported by these two special cases, is actually true for all cases, retinal discrimination combined with retinal adaptation accounts for nearly the whole of color constancy. That retinal processes do not wholly account for color constancy and the allied phenomena of color transformation, is proven by the work of Katz, Jaensch, Kroh and Gelb; but it is likely that the post-retinal effects extend the ability of the observer to judge object colors simply by extending the ability, already provided him to a considerable degree by retinal adaptation, to discount the color of the illuminant. Memory color, if it enters at all, need be invoked for only a very minor and unessential role.

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[14] A. Kohlrausch, *Tagessehen, Dämmersehen, Adaptation, I. Allgemeines über Umstimmung und "Farbenkonstanz der Sehdinge,"* in Handb. d. normalen u. pathologischen Physiol. **12**, 2d half, Receptionsorgane II, 1499–1506; Berlin, Springer (1931).

Chromaticity Sensibility to Wave-Length Difference as a Function of Purity*

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Using apparatus designed by Priest and Gibson, the writer has determined sensibility to wave-length difference for his right eye: (1) For stimuli of unit purity (spectral light) from 450 to 645 mμ. (2) For stimuli consisting of artificial noon sunlight plus homogeneous light of some selected wave-length, the wave-lengths being 455, 470, 481.5, 493, 530, 580, 635 mμ, and the purities ranging from unity to a few percent. (3) For stimuli consisting of homogeneous light of wave-length 455 plus some one of heterogeneous stimuli specified as follows: (a) equal energy, (b) color temperature 2570°K, (c) color temperature over 24,000°K. (4) For a stimulus consisting of homogeneous light of wave-length 530 mμ plus heterogeneous light of color temperature over 24,000°K. The results of (1) are concordant with previous determinations by others. The least perceptible difference (LPD) in wave-length for (2) shows, for wave-lengths 635, 580, 530 and 493 mμ at first a slow and then, near zero purity, a rapid increase as purity is decreased. For wave-length 455 mμ with decreasing purity, the LPD at first increases and then decreases to a pronounced minimum at about fifteen percent purity, increasing rapidly beyond this. Wave-lengths 470 and 481.5 mμ show a similar but less marked effect. The experiments noted under (3) and (4) were undertaken in order to study the effect of the spectral distribution of the heterogeneous stimulus in modifying the peculiar results just described for wave-lengths less than 482 mμ. It is shown that the form of the curve showing LPD as a function of purity depends upon the spectral distribution of the heterogeneous stimulus. Two hypotheses are proposed to account for the peculiar results for short wave-lengths.

I. INTRODUCTION

THE sensibility of the eye to change in wave-length of the stimulating radiant energy has been determined by a number of observers[1] for homogeneous radiant energy. The method has been in the majority of cases to present to the eye a two-part photometric field illuminated by spectral light of identical wave-length and then to shift the wave-length of the light in half the field until a just perceptible difference is observed. If the brightness of the two fields is kept equal, a precaution not always taken by the earlier workers, this shift in wave-length is the threshold or limen (called LPD hereafter) for wave-length change. The sensibility to wave-length change may, of course, be due either to hue or saturation difference[2] or to both.

The work described here is a study of the least perceptible difference in wave-length for stimuli of different purities; that is, for a double stimulus composed of a mixture of (1) homogeneous energy and (2) some one of several particular spectral distributions of energy, referred to below as heterogeneous stimuli. When this work was started, artificial noon sunlight was selected as a tentative heterogeneous stimulus to be used throughout the investigation. The results obtained with this stimulus and homogeneous radiation of wave-length 455 mμ, however, made it seem desirable to try the effect of substituting for it other heterogeneous stimuli. These others will be specified later. A like investigation by Watson[3] has been carried out for three wave-lengths and for purities ranging from one hundred to twenty-four or thirty-eight percent.

* Partial report on cooperative investigations in colorimetry by the Bureau of Standards and the Munsell Research Laboratory. This paper is a contribution to the program of determination of the visual psycho-physical functions (Priest: Appendix to B. S. Sci. Pap. 417, August, 1921) which is being carried on at the Bureau. Publication approved by the Director of the Bureau of Standards of the U. S. Department of Commerce and the Director of the Munsell Research Laboratory.

† Munsell Research Associate in Colorimetry at the National Bureau of Standards, June–September, 1926.

[1] König and Dieterici, Graefe's Arch. f. Opht. Abt. [2] **30**, 171 (1884). König, Arch. f. Anat. u. Physiol. 160 (1885). Uhthoff, Graefe's Arch. f. Opht., Abt. 4, **34**, 1 (1888). Brodhun, Zeits. f. Psych. u. Physiol. d. Sinnesorg. **3**, 97 (1892). Exner, Wien, Sitzb. 2A, **111**, 875 (1902). Steindler: Wien, Sitzb. 2A, **115**, 39 (1906). Jones, J. Opt. Soc. Am. **1**, 63 (1917). Laurens and Hamilton, Am. J. Physiol. **65**, 547 (1923). Hamilton and Laurens, Am. J. Physiol. **65**, 569 (1923).

[2] Troland, J. Opt. Soc. Am. and Rev. Sci. Inst. **6**, 544 (1922).

[3] Watson, Proc. Roy. Soc. **B84**, 118 (1911).

II. Apparatus

The essential feature of the apparatus[4] is a good photometric field, one part of which is illuminated by spectral light of constant wave-length and brightness plus a constant brightness of heterogeneous light, while the other part of the field is illuminated by the same brightness of the chosen heterogeneous light plus spectral light whose brightness may be kept equal to that of the spectral light in the first half of the field, while the wave-length may be changed continuously by known amounts.

A horizontal section of the apparatus is shown in Fig. 1. The following description is quoted by permission from an unpublished abstract by Priest and Gibson:

> The heart of the apparatus is a constant-deviation grating spectrometer (Hilger). The ordinary collimator slit has been replaced by a special slit (Fig. 2) divided at the middle of its length so that one-half of it may be displaced relative to the other half in a plane normal to the collimator, the direction of displacement being parallel to the plane of telescope and collimator. The observer looks through a slit in the focal plane of the telescope so that he sees the telescope objective in homogeneous light of one wave-length provided the two parts of the collimator slit are in line. A bi-prism at the telescope objective permits the observer to see a divided field, one

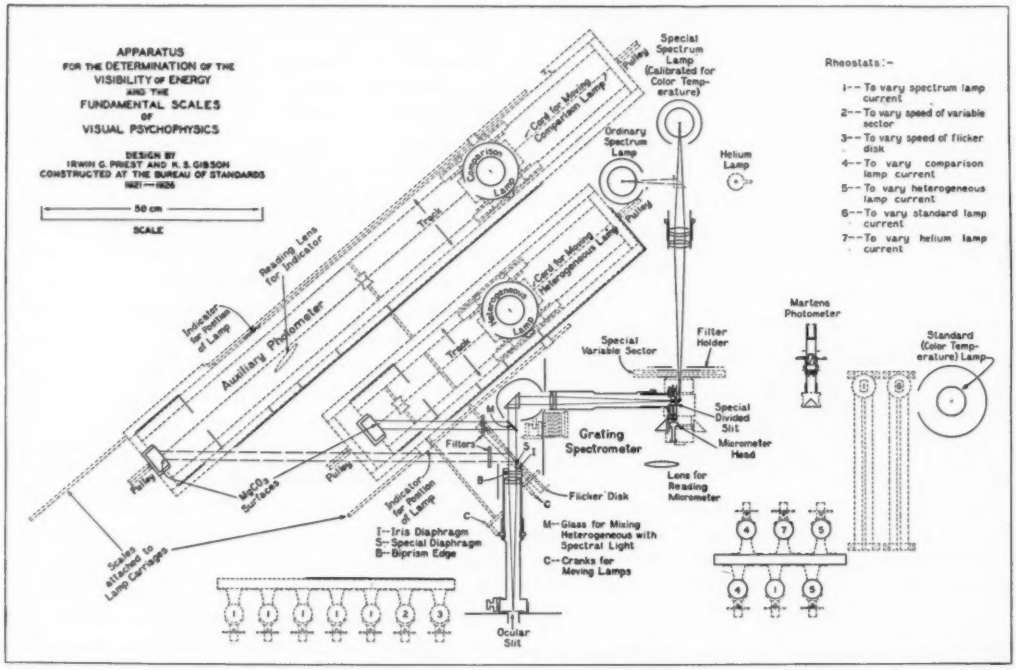

FIG. 1. Horizontal section of the apparatus.

part illuminated by light from the fixed slit, the other part by light from the movable slit.

A micrometer screw, reading directly in millimicrons, enables any small wave-length difference to be set by displacement of one part of the collimator slit relative to the other. The relative brightness of the two parts of this field may be varied by a special Keuffel and Esser sector photometer at the collimator slit.

Equal brightnesses of heterogeneous light are added to these two homogeneous fields by inserting a plate of clear glass in the spectral beam (between grating and telescope) at 45° to the axis of the telescope, so that the spectral light passes through the plate and the heterogeneous light is reflected from it in the same direction.

[4] The apparatus was designed and assembled by Irwin G. Priest and K. S. Gibson, J. Opt. Soc. Am. and Rev. Sci. Inst. **14**, 136 (1927).

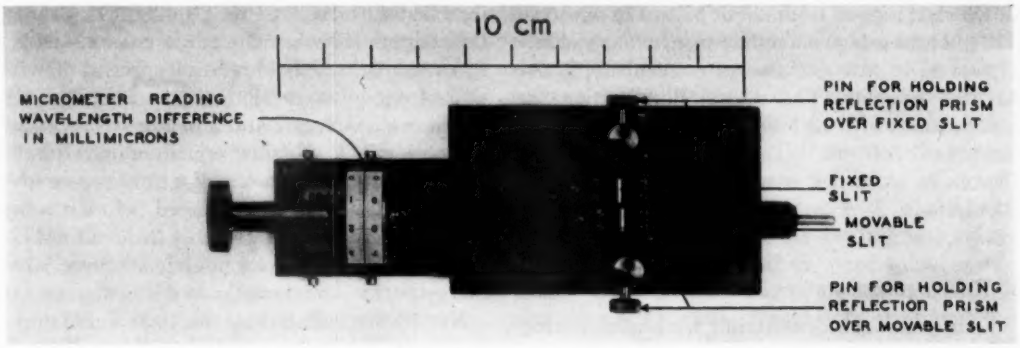

FIG. 2. Special collimator slit divided into fixed and movable halves. Designed by Irwin G. Priest.

The heterogeneous light is received from a magnesium carbonate block which is illuminated by a 1000-watt gas-filled lamp used in conjunction with a filter appropriate to give the desired color. This lamp is mounted on a track which provides for adjustment of brightness.

By means of an auxiliary photometer attached to the apparatus, the total field brightness as well as its homogeneous and heterogeneous components may be measured. This auxiliary photometer consists essentially of:

(1) A polished metal flicker disk (cut in quadrants) in front of the telescope objective, capable of rotation in a vertical plane at 45° to the axis of the telescope,

(2) A diffusing surface of magnesium carbonate so placed that light from it is reflected into the telescope by the polished sectors of the flicker disk.

(3) A lamp (known as the comparison lamp) on carriage and track, illuminating this diffusing surface.

III. CONDITIONS OF OBSERVATION

1. The observer's color sense

All observations were made with the observer's right eye and the following statements refer specifically to that eye. The observer falls within the group of approximately normal trichromats. As judged by the tests of Nagel[5] and Stilling,[6] he is normal. As judged by the anomaloscope test (Rayleigh equation)[7] he falls well within normal limits,[8] the slight departure from average normal being in the direction of deuteranomaly.[9]

The observer's visibility by the step-by-step, equality-of-brightness method, is on record.[10] He is somewhat red-sensitive.

2. Field brightness, artificial pupil, and retinal illumination

By measurements described below, field brightness was calibrated in terms of the scale of the comparison lamp. From this and the known size of artificial pupil, the retinal illumination may be computed.[11] The spectrometer ocular slit, which serves as artificial pupil, does not, of course, lie in the nodal plane of the eye, but the photon values given herein are computed from the field brightness as if it did. The true values would be only slightly different. The height of the ocular slit is 3.66 mm. Its width is variable. For all observations made between wave-lengths 510 and 645 mμ, it was 0.52 mm, while for wave-lengths less than 500 mμ, it was 1.56 mm. For most of the work here reported, a retinal illumination of 120 photons was adopted as standard. This is of the same order as that employed in the visibility investigation of Gibson and Tyndall,[12]

[5] *Tafeln zur Untersuchung des Farbenunterscheidungs-Vermögens*, W. Nagel, 7th Edition, Bergmann, Wiesbaden, 1911.

[6] *Pseudo-Isochromatische Tafeln zur Prüfung des Farbensinnes*, J. Stilling, 14th Edition, Leipzig, 1913.

[7] Lord Rayleigh, Nature, **25**, 64 (1881). Nagel, Zeits. f. Augenheilkunde **17**, 201 (1907). Rosmanit, *Anleitung zur Feststellung der Farbentüchtigkeit*, Leipzig, 1914.

[8] Rosmanit, reference 7, pp. 76–79.

[9] Tests by Priest at the Bureau of Standards, June–September, 1926.

[10] Gibson and Tyndall, B. S. Sci. Pap. No. 475, 156 (1923), Observer No. 1.

[11] The unit used is the photon as defined by Troland, Trans. Illum. Eng. Soc. **11**, 950 (see footnote 1) (1916).

[12] Reference 10, p. 142 and Fig. 22.

in which it ranged from about 85 to 420 photons; 120 photons is high enough to insure cone vision.[13] However, it was not always convenient to use so high a value. The retinal illumination for observations with variable purity at wave-lengths less than 500 mμ was mostly 100 photons, though in some few cases it dropped as low as 40 photons. Specific values are given with the results.

3. Heterogeneous stimuli

Four heterogeneous stimuli were used during the course of the investigation. Their energy distributions, all arbitrarily placed equal to unity at 560 mμ, are shown in Fig. 3.

No. 1 was produced by a 1000-watt lamp unfiltered. Its color temperature was 2570°K.

No. 2 was that used throughout most of the investigation. It was produced by the 1000-watt gas-filled lamp in conjunction with a single daylight filter. The voltage on the lamp was varied until the light transmitted by the filter was color matched with Abbot-Priest sunlight.[14] Although this stimulus evokes the same color as Abbot-Priest sunlight, it is in relative spectral distribution of energy somewhat different as shown by distribution curves 2 and 5 of Fig. 3.

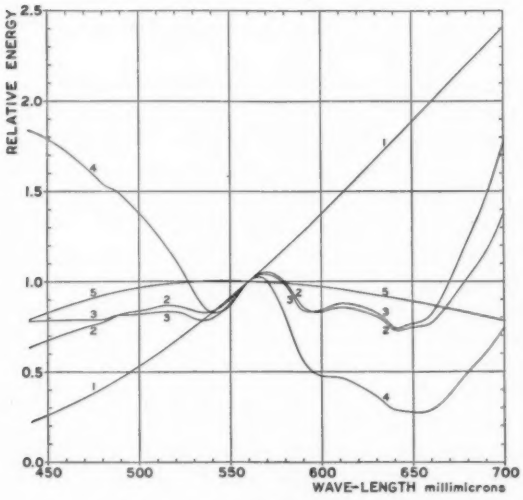

FIG. 3. Relative energy of heterogeneous stimuli. Curve 1. 1000-watt lamp alone, color temperature 2570°K. Curve 2. 1000-watt lamp and one daylight filter; a color match for Abbot-Priest sunlight. Curve 3. A color match for an equal-energy stimulus. Curve 4. 1000-watt lamp and two daylight filters, color temperature above 24,000°K. Curve 5. Abbot-Priest sunlight.

No. 3 was a so-called equal-energy stimulus. It was produced by means of a filter prepared by Gibson,[15] designed, when placed before a source whose energy distribution is that of Abbot-Priest sunlight, to color match a source whose energy is the same for all wave-lengths.

No. 4 was produced by the 1000-watt lamp in conjunction with two daylight filters. The color temperature is above 24,000°K. Its spectral centroid[16] is about 544.9 mμ.

4. Field

The field used was circular, divided along a horizontal diameter, and subtended 2° at the ocular slit. The surrounding field was dark.

IV. Calibrations

The wave-length scale of the spectrometer had been calibrated by a previous worker in this laboratory, and was simply checked from time to time with various helium lines.

To measure the brightness of the photometric field, a substitution method was used. A brightness standard was placed so that its brightness was substituted for the brightness usually produced either by the mixture of homogeneous and heterogeneous stimuli, or by each stimulus separately. This known brightness could thus be balanced against the brightness due to the comparison lamp by finding the scale reading of the latter for a photometric match, i.e., zero flicker. This gave the calibration of one point of the comparison-lamp scale in actual brightness. The field brightness due to the mixture or to each component was then determined in terms of this scale by using the inverse square law. The validity of the inverse square law for use with both the comparison lamp and the 1000-watt lamp had been carefully checked by previous workers

[13] Troland, Trans. Illum. Eng. Soc. **11**, 955 (1916), gives 60 photons as a lower limit.

[14] Priest, J. Opt. Soc. Am. and Rev. Sci. Inst. **12**, 479 (1926). Judd, B. S. J. Research **4**, 524 (1930).

[15] Gibson, J. Opt. Soc. Am. and Rev. Sci. Inst. **13**, 305 (1926).

[16] Priest, J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 1184 (1923).

in this laboratory.[17] To compute the retinal illumination, corrections were made for loss of light due to reflection by (1) telescope lens, (2) telescope bi-prism, (3) observer's spectacles. This gave, for comparison lamp at scale reading 90, 4.07 millilamberts, which is 12.95 photons when viewed through a one square millimeter pupil.

The method of rotatory dispersion[18] was employed to obtain the color temperature of the 1000-watt lamp. Two observers of normal color vision, Dr. K. S. Gibson and the writer, made the color-matching observations.[19] The other relative energy curves (except curve 5 of Fig. 3) were obtained by computation from the relative energy of the unfiltered lamp and the known transmissions of the filters used.

V. Method of Making Observations

For unit purity the comparison lamp was set at the correct position for the desired retinal illumination and the current in the spectral lamp varied until equality was obtained, using the flicker disk. The comparison lamp was then extinguished and flicker disk stopped with an open sector in the line of sight. The K. and E. sector was then started. Allowance for the reduced retinal illumination with the K. and E. sector running had, of course, been made in the first setting of the comparison lamp. The screw which operated the movable slit was now turned slowly until there appeared a perceptible difference in the two parts of the field. If a readjustment to equality of brightness by means of the sectored disk caused this difference to disappear, the slit screw was again turned slowly. If, when matched in brightness, the two parts of the field were definitely different, the reading was taken as an LPD. These readings were taken alternately with displacements toward longer (R) and shorter (V) wave-lengths. In the curve for unit purity each plotted point is the average of at least five R and five V settings, while for lesser purities ten R and ten V settings were always taken.

For purities less than unity, the comparison lamp was set first at a position to give the desired brightness due to the heterogeneous stimulus and the correct position for the 1000-watt lamp was determined. The comparison lamp was then set at such a position as to give the total desired brightness and the spectral lamp was then lighted (the K. and E. sector not running) and its current varied until it and the 1000-watt lamp, in the position just determined, balanced the comparison lamp approximately. The current was then kept steady at this value and a set of observations made on the position of the comparison lamp. The 1000-watt lamp was then extinguished and the comparison lamp set to balance the spectral lamp. From the appropriate readings of the comparison lamp scale the brightness of heterogeneous light, H, the total brightness, T, and brightness of spectral light, S, could be computed. The purity, p, is then: $p=S/T$, or $p=1-H/T$. The comparison lamp was then extinguished, the sector started, and the 1000-watt lamp moved back on the scale to reduce the brightness of the heterogeneous component in the same proportion that the sector reduced the brightness of the spectral component. The LPD was then determined as described above.

VI. Results and Discussion

1. Unit purity

The observations at unit purity were made primarily to locate the points of maximum and

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[17] K. S. Gibson and F. G. Brickwedde.

[18] Priest, J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 1175 (1923).

[19] It should be mentioned that the 1000-watt lamp did not stay constant throughout its use. With the daylight filter it was color-matched, as stated before, at the start of this work with Abbot-Priest sunlight, approximately equivalent to a color temperature of 5325°K, but at the completion of the experimental work, its color temperature with the same filter was found to be 5820°K, a rise of about 500°K. The color temperature of the lamp alone was found at this time to be 2650°K. By using the spectral centroid relation, Gibson, J. Opt. Soc. Am. and Rev. Sci. Inst. **11**, 473 (1925), the original color temperature of the lamp alone was computed to be 2570°K, and it is the relative energy for this temperature which is given by curve 1 in Fig. 3. The reason for using the initial rather than the final distribution of energy of the lamp is that the parallel change of brightness, of which a continuous record was made in terms of the comparison lamp, was very small during the period that the heterogeneous stimulus matching Abbot-Priest sunlight was used. The exact specification of the other heterogeneous stimuli is not important.

minimum sensibility of the observer's eye, since it had been planned to make the observations at variable purities at these points first. Fig. 4 is a plot of the results obtained. This curve shows as usual, in the range of wave-lengths taken, three maxima and three minima. A fourth minimum

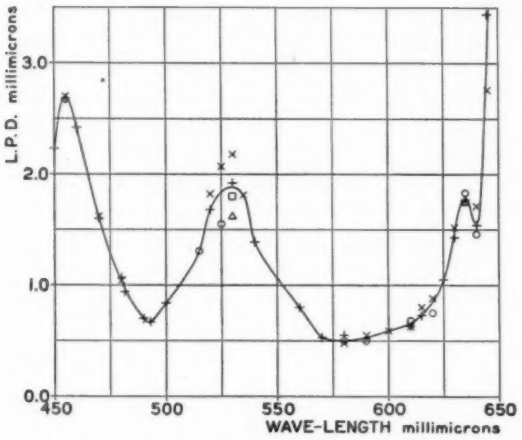

FIG. 4. Least perceptible difference for unit purity.

LPD lies below 450 mμ. The retinal illumination was 120 photons from 625 to 490 mμ; from 625 to 645 mμ the illumination varied from 120 to 60 photons; from 490 to 460 mμ it decreased from 120 to 60 photons. The observations at 455 mμ were taken at 40 photons, and at 450 mμ at a somewhat lower illumination. For wave-lengths less than 490 mμ a stray light filter was placed over both halves of the spectrometer collimator slit. The collimator slit was fixed in width. This width was such as to include a wave-length range of 3.0 mμ throughout the spectrum. From 510 to 645 mμ the ocular slit was 0.52 mm (a range of 3.0 mμ) and from 450 to 500 mμ it was increased in width to 1.56 mm (a range of 9 mμ).

The LPD curve (Fig. 4), except between 520 and 540 mμ, is very definite and reproducible. This is shown by the good agreement between different observations at the same wave-length some of which were taken several weeks apart. The sequence in which observations were taken did not seem to be a determining factor. Some of the points shown were taken in a long-to-short wave-length sequence, some in the reverse order, and others yet were isolated observations. From 515 to 540 mμ, however, the points are somewhat scattered. The sequence in this region was as follows: points designated by crosses (+), 520, 530, 540; circles (o), 515, 525; *x*'s (*x*), 520, 525, 530, 535. All these readings were taken on the same day and consecutively. The two other points at 530 mμ (squares, triangles) were isolated readings taken during variable purity determinations at this wave-length. Laurens and Hamilton[20] report an anomaly in this region depending on the order in which the readings are taken.

2. Variable purity, heterogeneous stimulus matching Abbot-Priest sunlight

In Fig. 5 are curves showing the relation between LPD and purity, each for a single wave-length, and, with the exception of the curve marked 530 *B*, for the heterogeneous stimulus

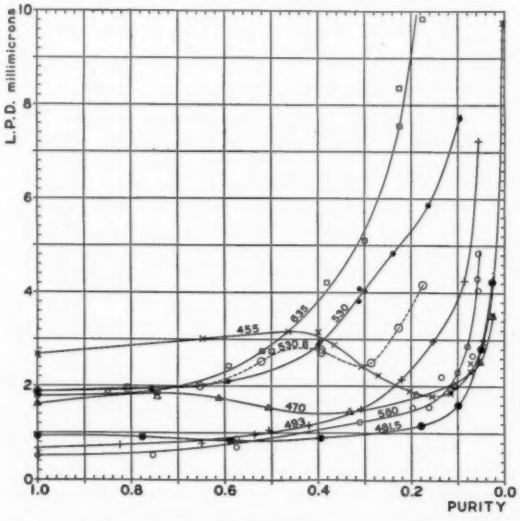

FIG. 5. Least perceptible difference as a function of purity. The heterogeneous stimulus has the energy distribution of curve 2, Fig. 3, except for 530*B* for which it is that of curve 4, Fig. 3. Wave-length of the homogeneous stimulus is indicated on curves.

which color matches Abbot-Priest sunlight (curve No. 2, Fig. 3). For wave-lengths 635, 580, 530 and 493 mμ, the LPD increases with decrease of purity. This is similar to the result found by Watson. The four curves, moreover, are much

[20] Reference 1, p. 559.

alike. This similarity between curves for different wave-lengths is better exhibited in Fig. 6, where the ratio of the LPD at any purity to the LPD at unit purity is plotted against purity for each one of the four wave-lengths. The correspondence shown makes it possible to determine roughly

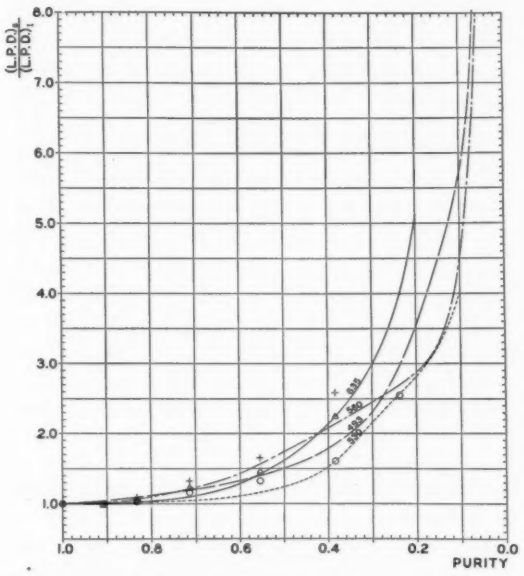

FIG. 6. Relative least perceptible difference. Wave-lengths are marked on curves. Watson's results are shown by: circles, wave-length 589 mμ, plus signs, wave-length 632 mμ, triangles, wave-length 527 mμ.

from the LPD at unit purity, the LPD at any wave-length between 493 and 635 mμ for any purity, assuming, what seems justifiable, that the intermediate wave-lengths act like these four. Watson's results, reduced in the same way, are plotted in this figure also. He does not specify his heterogeneous stimulus other than as a "white light."

It was originally intended to take all four of these curves at a constant retinal illumination of 120 photons. Owing to a miscalculation most of the points on the 580 and 530 curves for purities less than unity were taken at illuminations ranging from 120 to 250 photons. When this error was discovered, enough new observations were made at the standard illumination to make sure that the course of the curves was unaffected by the higher illumination. The 635 and 493 curves were taken at 120 photons, except for 635, unit purity, which was 85 photons.

The LPD-purity curve for 455 mμ (Fig. 5) shows at first an increasing LPD as purity is decreased and then a surprising decrease to a minimum at 15 percent purity. At still lower purities the curve rises again. At wave-length 470 mμ (Fig. 5) the curve is of the same type, though the minimum is much less marked. At 481.5 mμ (Fig. 5) the minimum has almost disappeared. A possible explanation of this behavior lies in assuming that the heterogeneous stimulus employed is not a true subjective neutral[21] for the observer. The addition of such a pseudo neutral in increasing proportions to homogeneous radiant energy of wave-length 455 mμ would gradually shift the dominant wave-length of the mixture away from 455 mμ when expressed in terms of the true neutral. This shift must be toward a wave-length for which the LPD at unit purity is smaller since a maximum LPD for unit purity occurs at 455 mμ. Hence the LPD measured for 455 mμ and the actual heterogeneous stimulus used would also be lowered in value since its changes (which are actually measured) must parallel more or less closely the LPD for the hypothetical dominant wave-lengths and true subjective neutral. If it is assumed that the behavior shown in Fig. 6 for a tentative neutral, which is perhaps not very different from the true neutral, is typical of the whole spectrum for the true neutral then the observed curve at 455 mμ is the resultant of two opposing effects, the first of which increases the LPD with decreasing purity while the second shifts the dominant wave-length to values for which the LPD at unit purity is smaller. This shift increases with the quantity of heterogeneous stimulus added until the second effect overbalances the first for a small range of purities. Obviously the first effect must be predominant at very low purities. This same explanation would apply also in the case of 470 and 481.5 mμ, the minimum LPD being less pronounced as the wave-length increases because the LPD at unit purity is smaller.

Another possible explanation of the anomaly observed for wave-lengths less than 493 mμ

[21] Priest, J. Opt. Soc. Am. and Rev. Sci. Inst. **13**, 306 (1926).

would be to attribute it entirely to a saturation effect. Thus one might assume that a given wave-length shift for a purity near unity and therefore for high spectral brightness, would introduce less saturation difference than the same wave-length shift at some low purity with its accompanying low spectral brightness in spite of the fact that the total brightness is the same for the two cases. Indeed if the assumption is made that the variation of saturation with brightness for spectral light is independent of the amount of heterogeneous light mixed with the spectral light, then results by Haupt[22] substantiated by Purdy[23] afford some support to this hypothesis. Haupt and Purdy have shown that in some regions of the spectrum, starting with very low brightness, the saturation first increases with brightness to a maximum and then decreases as the brightness reaches higher values. Thus saturation differences between neighboring wave-lengths are lessened at brightnesses considerably above the brightness for maximum saturation. The brightness of maximum saturation is lower in the blue and violet than in the rest of the spectrum.

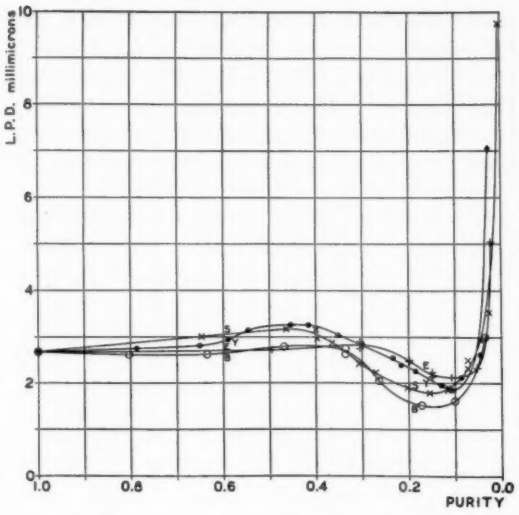

FIG. 7. Effect of varying the heterogeneous stimulus for wave-length 455 mμ. The heterogeneous stimuli are as follows: *S*, artificial noon sunlight, curve 2, Fig. 3. *Y*, color temperature 2570°K, curve 1, Fig. 3. *B*, color temperature above 24,000°K, curve 4, Fig. 3. *E*, "equal energy," curve 3, Fig. 3.

3. Variable purity, other heterogeneous stimuli

The data shown graphically in Fig. 7 were taken in order to decide, if possible, between these two hypotheses. Four curves are shown, each for the same homogeneous stimulus combined with some one of the heterogeneous stimuli previously described. The curve marked "*S*" is curve 455 of Fig. 5. The curve marked "*E*" is for heterogeneous stimulus No. 3. In this, as the filter previously described was of too small an aperture to be placed directly in the beam from the 1000-watt lamp, it was used over the ocular slit. This had the disadvantage of reducing the intensity of the spectral light, so that the retinal illumination for the 50 and 30 percent purities (curve *E*, Fig. 7) are about 45 and 75 photons, respectively. All the other points are for about 100 photons. Curve *Y*, Fig. 7, refers to heterogeneous stimulus No. 1, while curve *B*, Fig. 7, is for heterogeneous stimulus No. 4. In curve *B* the retinal illumination is between 60 and 95 photons; in curve *Y*, mostly 100 photons, though for some of the higher purities it is about 60 to 80 photons.

The relative positions of the *E* and *S* curves seem to support the first hypothesis, since such a slight change in the quality of the light should not affect saturation effects appreciably. If this be so, it is interesting to note that the true subjective neutral according to the first hypothesis is, for the writer at least, nearer to an equal-energy distribution (purple as compared with sunlight) than to the artificial noon sunlight.[24] This suggests a method for finding the true neutral stimulus; that is, define it as that heterogeneous stimulus which will make all the curves of Fig. 5 of the same type. This type will probably be that already described for the rest of the spectrum. Curves *Y* and *B* are undoubtedly partially explainable by the first hypothesis.

The first hypothesis is also supported by experimental results shown by curve 530 *B*, Fig. 5, in which heterogeneous stimulus No. 4 was used for a set of observations at wave-length 530 mμ. This wave-length was chosen because it gives a

[22] Haupt, J. Exp. Psych. **5**, 347 (1922).
[23] Purdy, Brit. J. Psych. **21**, part 3, 283 (1931).
[24] Cf. Priest, J. Opt. Soc. Am. and Rev. Sci. Inst. **13**, 306 (1926).

maximum value of the LPD at unit purity and because of the probable absence of the saturation effect. The maximum LPD is not as great, however, as that at 455 mμ and, therefore, the effect shown by curve 530 *B* is not as marked as it is at 455 mμ. It is concluded that the first hypothesis is really suitable to explain a decrease in the LPD with decreasing purity. Since, however, no heterogeneous stimulus has been found which eliminates the anomalous minimum, it is impossible, judging from these data alone, to make a definite choice between the two hypotheses advanced above.

It is worth noting, however, that the recent empirical discovery by Judd[25] of a general expression for chromaticity sensibility favors the second hypothesis over the first. This expression yields the anomaly for the same wave-length regions for which it was found experimentally, and gives a fair quantitative agreement for all wave-lengths. Examination of the empirical expression in order to discover with what factor the anomaly is related shows that it is linked with the relatively low luminosity coefficient for violet. Since it is probable that the low luminosity coefficient for violet compared to that for red or for green is connected with the saturation effects for blue and violet studied by Haupt and Purdy, the second hypothesis, rather than the first, would appear to touch upon the true explanation of the anomaly for Abbot-Priest sunlight (or its color equivalent) as heterogeneous stimulus. It is doubtful, in view of this substantiation of the second hypothesis, whether any heterogeneous stimulus exists which would make all the LPD-purity curves of the same shape. But the first hypothesis may contribute somewhat to the anomalous minimum; it cannot be entirely rejected because these experiments have shown that the shape of the LPD-purity curve depends on the selection of heterogeneous stimulus.

VII. Accuracy

It should be mentioned that a few sets of readings (10 in all for the curves of Figs. 5 and 7) were discarded as erratic. These were nearly always too high and in many cases were obtained when eye fatigue was certainly present. Although the eyes were rested at frequent intervals fatigue was noticeable, particularly in the late afternoon.

With these exceptions, the agreement between individual settings for the LPD was surprisingly good when it is considered that a threshold quantity is not very definite. To illustrate better the degree of agreement obtained, the probable errors of some of the observations from which the curves of Fig. 7 are plotted have been computed. These points have been chosen because the curves of this figure are close together and doubt might well arise as to whether they are definitely distinct within the experimental error. These computed errors are given in Table I. Each value is a combination of the separate probable errors for the ten *R* and ten *V* settings.

Table I.

Curve	Percent purity	LPD mμ	Probable error, mμ
S	15.5	1.78	±0.074
S	15.5	1.78_5	±.071
S	36.5	2.88	±.083
E	10.8	2.12	±.060
E	15.0	2.14	±.059
E	15.0	2.23	±.056
Y	18.6	2.26	±.054
Y	10.5	1.85	±.053
B	17.2	1.52	±.061
B	33.6	2.75	±.090
B	33.6	2.63	±.058

It may also be mentioned that the average of ten *R* and the average of ten *V* displacements are not always equal. In most cases the difference is doubtless accidental, but at 635 mμ for purities less than 50 percent (see Fig. 5) the average *R* displacements are all definitely larger than the average *V* displacements. That this should be so is apparent from a consideration of the shape of the curve of Fig. 4 in the neighborhood of 635 mμ. After the minimum at 640 mμ has been passed, the *R* displacements which produce observable differences occur at wave-lengths corresponding to very much decreased sensibility to wave-length change. The *V* displacements, on the other hand, are always towards a region of increased sensibility. Of all the other curves of Figs. 5 and 7, 455 *S* and 455 *Y* alone show a decided predominance of one over the other; and here the *V* displacements are larger. This may indicate, according to the first hypothe-

[25] Judd, J. Opt. Soc. Am. **22**, 72 (1932).

sis, a shift of dominant wave-length towards the red if the result could be expressed in terms of the true neutral; or it may indicate, according to the second hypothesis, that the saturation effects are changing more rapidly in the R direction than in the V direction.

VIII. Conclusion

For most of the visible spectrum (wave-length greater than 490 mμ) decreasing sensibility to wave-length change is found with decreasing purity of the stimulus. The curves of least perceptible difference against purity are similar in shape, though it is shown that a change in composition of the heterogeneous stimulus is able to change one of the curves considerably.

For the blue end of the spectrum an anomaly exists; the least perceptible difference goes through a minimum value as purity is decreased. The effect on this anomaly of varying the heterogeneous stimulus is shown. To explain this anomaly two qualitative and, perhaps, highly speculative hypotheses are advanced. The evidence deduced shows that the first of these hypotheses is able to produce an effect of the kind observed, though its magnitude is not predictable. There is also evidence in favor of the second hypothesis. It is hoped that these speculations may at least be of some use in shaping the course of further experiment.

In conclusion the writer wishes to express his indebtedness to the late Mr. Irwin G. Priest, who initiated this work and placed at the writer's disposal a completed and partially calibrated apparatus for its prosecution. Acknowledgment is also due Dr. D. B. Judd for much stimulating discussion in interpreting the results.

The Dependency of Visual Acuity upon Stimulus-Distance

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(Received October 13, 1932)

ALTHOUGH visual acuity as a factor in seeing has often been so over-emphasized as to unduly distract attention from other important factors, it is one of the fundamental functions from the viewpoint of physiologic optics. Probably its greatest importance lies in the perception of details within arm's reach. However, accurate measurements are more easily made at greater distances, because this avoids inherent difficulties in the construction and illumination of extremely small test-objects which are necessary to measure visual acuity at short distances.

During our researches upon the relationships of various external factors of threshold visibility, such as size, contrast, brightness and time of exposure, we have had various indications that visual acuity depends somewhat upon the distance between the object and the eyes. A serious investigation of this matter has been deferred from time to time but the recent researches by Freeman[1] indicate that the effect of stimulus-distance is considerable. They also indicate the necessity of investigating the effect with different test-objects and at definite levels of brightness, particularly those resulting from the higher intensities of illumination now known to be desirable.

Previous interpretations[2] of visual acuity data, for the purpose of establishing optimum conditions for seeing, do not necessarily require revision because of the fact that stimulus-distance was not included as a factor. In these interpretations, visual acuity data are generally viewed in a relative sense because it is well known that absolute visual acuity depends upon the character of the test-object. However, it is desirable to verify Freeman's results for other conditions and to establish the magnitudes of the factors necessary for transforming data obtained at one distance into data applicable to any other distance.

It is obviously essential to determine whether the relationship between stimulus-distance and visual acuity represents a true anomaly or an artifact arising from uncontrolled factors in the experimental procedure. The researches and analyses of Freeman offer no experimental evidence in support of the latter assumption. It is the purpose of this paper to present additional data bearing upon the point in question; and also, to extend the investigation of this anomaly of visual acuity beyond the near-threshold brightnesses used in previous researches.

Apparatus and Methods

An important characteristic of the present research is that measurements of visual acuity were made under normal or customary seeing-conditions insofar as was possible. The subjects were seated comfortably at a table fitted with a head-rest and viewed the test-object binocularly. The acuity test-objects were adjusted by means of a hand-wheel located conveniently at the right hand of the subject. The mechanical features of the apparatus were such that the test-objects could be altered in size smoothly, easily and quietly. The subjects were permitted as much time as desired for each setting of the apparatus and were particularly instructed to be certain that they actually perceived the details of the test-object.

In this work it was impractical to rotate the test-object into various positions as we have done in our more exhaustive previous investigations.[3] It is recognized that this experimental technique has the disadvantage of not providing

[1] Ellis Freeman, *Untersuchungen uber das indirekte Sehen*, Psychologische Forschung, **14**, Nos. 3 and 4 (1931).

Ellis Freeman, *Anomalies of Peripheral Visual Acuity*, J. Ex. Psy. **12**, 324 (1929).

Ellis Freeman, *Anomalies of Visual Acuity in Relation to Intensity of Illumination*, Am. J. Psy. **42**, 287 (1930).

Ellis Freeman, *Anomalies of Visual Acuity in Relation to Stimulus-Distance*, J.O.S.A. **22**, 285 (1932).

[2] M. Luckiesh and Frank K. Moss, *Seeing—A Partnership of Lighting and Vision*, Williams and Wilkins Co., Baltimore, 1931.

[3] P. W. Cobb and Frank K. Moss, *The Four Variables of the Visual Threshold*, J. Franklin Inst. **205**, 831 (1928).

a check upon the threshold values of visual acuity established by the subjects. A more rigid method of experimentation would require the subjects to report correctly upon the position of some critical detail of the object. However, the latter method usually involves many long series of tedious measurements. Since the absolute values of visual acuity are not comparable in importance with relative values, the method used is considered adequate for the purposes of this research.

The selection of a visual acuity test-object is an arbitrary but by no means an indifferent matter, as is shown later. It is obvious, for example, that the factor of irradiation must be considered in objects of high brightness, and that objects which are altered in form by changes in size complicate the analysis of the experimental results. Since no one type of test-object seemed satisfactory when appraised from both practical and theoretical bases, measurements were made employing two fundamentally different kinds of test-objects. A comparison of the results of parallel experiments with different test-objects also indicates the influence of the character of the test-object upon the existence of the anomaly.

Fig. 1 illustrates one of the two types of test-objects used and is designated in this discussion as the "parallel-bar" test-object. It consists of two white bars on a black background and separated by a distance equal to the width of the bars. The width of the bars, therefore, becomes the nominal dimension of the object. The length of the bars is three times the width of each and the entire pattern may be described as a white square with a strip constituting its middle third removed. The elements of this object consist of thin sheets of black and white papers mounted with precision on the six parallel-lever mechanisms shown in Fig. 1. Thus the object is alterable in size without changing the geometrical proportions of the pattern. Obviously, the design of this instrument, by Dr. P. W. Cobb, a former colleague, is such as to require the services of a skilled instrument-maker for its construction. Although this type of test-object possesses many obvious advantages, it is not entirely satisfactory for an investigation of the effect of stimulus-distance when near visual work is required due to shadows cast by the thin leaves which form the object. However, for distances greater than one meter the inherent errors due to construction become negligible and the shadows disappear with the decreasing obliquity of the illumination.

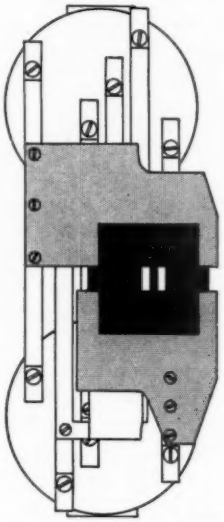

FIG. 1. The rotation of the upper disk produces corresponding changes in the parallel-lever mechanism to which the elements of the test-object are attached. This parallel-bar test-object is thus alterable in size without changes in geometrical form.

The second form of test-object used in this research is illustrated in Fig. 2 and is designated

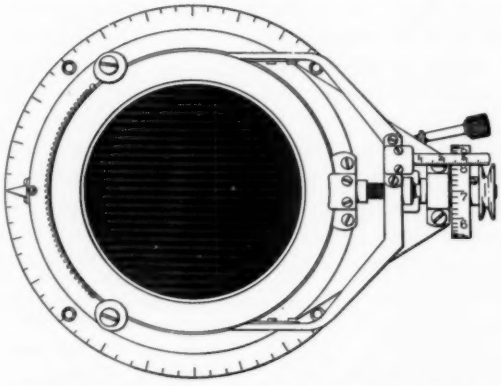

FIG. 2. The rotation in opposite directions of two glass plates ruled with parallel opaque lines produces dark bands across the face of the test-object. As the angular displacement between the two sets of parallel-lines is increased the number of dark bands increases and the width of the bands decreases. In this illustration, the number of parallel-lines is less than in the actual apparatus for reasons of simplicity and clearness.

as the "parallel-line" test-object. It consists essentially of two clear glass plates uniformly ruled with parallel opaque lines and was originally suggested by Dr. H. E. Ives, a former colleague.[4] The plates are arranged to be rotated about a common axis, thus varying the angular displacement between the two sets of parallel lines. When viewed at the proper distance and against a luminous background, the intersections of the two sets of parallel lines form dark bands, as is illustrated in Fig. 2. The number of bands increases and the bands grow smaller as the glass plates are rotated from a parallel position of the lines. The transmission-factor of the test-object remains constant. In the actual instrument, the opaque rulings are spaced 240 to the inch and hence it is possible to form very fine and adequately defined dark bands. The separation between bands is read in millimeters from a graduated micrometer screw.

These test-objects were centrally located behind a large white vertical screen and viewed through small square openings. The white screen was illuminated to brightnesses of 1 and 10 millilamberts, respectively, by a totally indirect lighting system which also provided general illumination for the entire room. The eyes were adapted to these brightness conditions before measurements were taken. The luminous background for the parallel-line test-object was maintained at these brightness-levels by an independent lighting arrangement.

Results

Visual acuity measurements were made at brightnesses of 1 and 10 millilamberts for distances of 60, 120 and 280 centimeters, respectively, with the parallel-line test-object; and at a brightness of 1 millilambert for distances of 120 and 280 centimeters with the parallel-bar test-object. A series of 5 measurements of visual acuity were made for each of 10 subjects upon 10 different days for each condition. Hence, each of the mean values of visual acuity presented in the tables represents an average of 500 individual measurements.

The relationships between visual acuity and stimulus-distance for the parallel-line test-object are summarized in Table I and graphically presented in Fig. 3.

Table I. *The relationship between visual acuity and stimulus-distance as determined with the parallel-line test-object.*

	Visual acuity*					
	Brightness = 1 ml Distance in cm			Brightness = 10 ml Distance in cm		
Subject	60	120	280	60	120	280
1	0.77	0.94	1.25	0.90	1.08	1.43
2	0.73	0.89	1.00	0.93	1.10	1.27
3	0.47	0.53	0.67	0.49	0.53	0.68
4	0.86	1.07	1.23	0.96	1.21	1.45
5	0.55	0.74	1.10	0.60	0.73	1.15
6	0.51	0.70	0.83	0.55	0.78	0.85
7	0.88	1.06	1.16	—	1.25	1.33
8	0.60	0.71	0.99	0.71	0.84	1.13
9	0.68	0.87	1.14	0.80	1.08	1.37
10	0.65	0.70	0.78	0.74	0.81	0.88
Mean	0.66	0.80	1.00	0.72	0.91	1.12
Probable error	0.006	0.008	0.010	0.005	0.008	0.010

* Visual acuity is defined as the reciprocal of the visual angle in minutes subtended by the critical detail of the test-object.

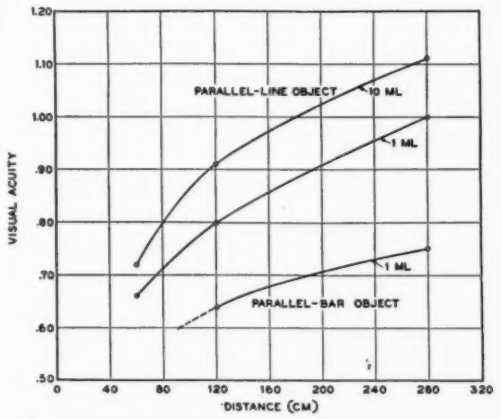

Fig. 3. The relationships between visual acuity and stimulus-distance for the parallel-line and parallel-bar test-objects.

It will be noted that visual acuity (reciprocal of the visual angle in minutes subtended by the critical detail of the test-object) is definitely a function of stimulus-distance. It is particularly significant that each of the ten subjects reacted in a similar manner. Since the probable errors of these data are generally less than 1 percent,

[4] H. E. Ives, *An Improved Visual Acuity Test-Object*, J.O.S.A. 1, 101 (1917).

it is evident that the differences in visual acuity for various distances are real. These results are in conformity with the results obtained by Freeman and in addition they establish the existence of the anomaly for binocular vision for work-world brightnesses and for objects requiring resolution.

It is evident from a casual inspection of the parallel-bar object (Fig. 1) and the parallel-line object (Fig. 2) that the critical details of the latter are the more easily seen because of the multiplicity of the lines. This observation is corroborated by the experimental results summarized in Table II.

TABLE II. *The relationship between visual acuity and stimulus-distance as determined with the parallel-bar test-object.*

Subject	Distance in cm 120	280
1	0.70	0.86
2	0.66	0.72
3	0.53	0.68
4	0.73	0.79
5	0.66	0.75
6	0.74	0.92
7	0.57	0.66
8	0.61	0.69
9	0.62	0.78
10	0.64	0.68
Mean	0.64	0.75
Probable error	0.006	0.006

It will be noted from these data that the influence of stimulus-distance upon visual acuity is similar for the two types of test-objects although the absolute values of visual acuity are distinctly different. These results emphasize the fact that visual acuity measurements possess meaning only in the relative sense. Since one of these objects consisted of many opaque lines upon a luminous background which were alterable in width but not in length and the other consisted of white bars upon a black background and simultaneously alterable in all dimensions, it may be concluded that the existence of the anomaly does not depend upon the character of the test-object.

THEORETICAL CONSIDERATIONS

The size of the pupil is a factor to be considered in the discussion of this visual anomaly, since it varies with accommodation. The diameter of the pupil for each experimental condition is presented in Table III.

Each value in the table represents the average of five series of three measurements each for each of the ten subjects. By reference to Fig. 3 it is obvious that the changes in retinal brightness due to changes in pupil size can account for only a small fraction of the differences in visual acuity which were observed at various distances.

TABLE III. *The relationship between pupil size and stimulus-distance.*

Subject	Diameter of pupil in millimeters Brightness = 1 ml Distance in cm 60	120	280	Brightness = 10 ml Distance in cm 60	120	280
1	4.57	4.98	5.24	3.16	3.65	3.72
2	6.53	6.71	6.81	4.65	4.66	4.72
3	6.10	6.17	6.43	3.89	3.93	3.93
4	6.03	6.28	6.31	4.23	4.48	4.42
5	3.88	4.25	4.58	3.11	3.10	3.42
6	7.06	7.31	7.41	5.16	5.14	5.18
7	4.30	4.79	4.82	—	3.26	3.29
8	6.24	6.40	6.41	4.69	4.82	4.88
9	5.84	6.06	6.10	3.26	3.36	3.38
10	3.13	3.22	3.23	2.43	2.48	2.51
Mean	5.37	5.72	5.73	3.84	3.89	3.94

It has been suggested[1] that the crystalline lens may be a more efficient refracting instrument when it is thin than when it is relatively thick. It is also conceivable that the image upon the retina may be steadier when the eyes are fixated for distant vision than when they are for nearer vision. A discussion of the data from this viewpoint follows.

An analysis of the data by subjects indicates a large variation in the magnitude of the effect of stimulus-distance upon visual acuity. The greatest individual increase in visual acuity due to a change in distance from 60 to 280 cm is 96 percent; the least is 20 percent; and the average is 54 percent. These results represent averages of the data obtained at both brightness-levels. A comparison of data for individual subjects presented in Tables I and III indicates a correlation between influence of stimulus-distance and changes in pupil size due to changes in distance. A statistical analysis of the results gives a positive coefficient of correlation between changes in visual acuity and pupil size of 0.35 with a corresponding probable error of 0.18. Such a coefficient

of correlation is admittedly small in comparison with its probable error. However, it suggests that an explanation of this anomaly may be found in the functioning of the mechanism of accommodation, since the size of the pupil is a function of accommodation and shows some correlation with the stimulus-distance anomaly of visual acuity.

Summary

Data pertaining to the influence of stimulus-distance upon visual acuity have been extended to include (1) everyday seeing conditions, (2) binocular vision, (3) two radically different test-objects; and (4) they provide a basis for transforming existing visual-acuity data from one stimulus-distance to another within a wide range.

The authors acknowledge the assistance of Mr. S. K. Guth in the accumulation of these data and in the computation of the results.

A Triple Interferometer for Distinguishing Flexural and Longitudinal Vibrations in Quartz

H. OSTERBERG, *Department of Physics, University of Wisconsin*

(Received September 29, 1932)

A triple interferometer arrangement for determining the relative motion of opposing areas on the front and back surfaces of any vibrating plate is described and illustrated. The manner in which this interferometer is used to distinguish between flexural and longitudinal modes in quartz plates is also discussed and illustrated. These illustrations are selected to demonstrate several very interesting characteristics of an oscillating quartz plate. In the case of a quartz cube operated as an X-cut the influence of lateral vibrations is seen to affect seriously the computed frequency of the fundamental longitudinal waves propagated along X. A simple interferometer employed in the study of quartz plates is described, together with a strip method of excitation used in conjunction with this interferometer.

THE discoveries of piezoelectricity and magnetostriction, together with their converse effects, have led to convenient methods for creating sustained vibrations of various types and of wide range both in frequency and amplitude in an appropriate elastic solid. The longitudinal and flexural modes are of particular importance on account of their practical and theoretical importance. More systematic and accurate observations upon these modes are needed to support further investigations into the theory of elastic solids.

The interferometer method of examining quartz plates possesses the following important advantages over the dust and glow methods: (1) It leaves no uncertainty, even at a casual glance, regarding the location of the nodes and antinodes; (2) it can be modified to determine the relative motion of corresponding areas on opposing surfaces; (3) it measures the amplitude of oscillation; (4) it provides the greatest speed and convenience of observation when the technique of adjustment has been mastered. Contrary to common beliefs, an interferometer need not be expensive.

Mention[1] has been made that the flexural and longitudinal modes can be distinguished by a triple interferometer arrangement in which the two returning mirrors are: (1) the front and back surfaces of the vibrating plate; (2) the front surface and an auxiliary mirror; (3) the back surface and an auxiliary mirror.

[1] H. Osterberg, J.O.S.A. **22**, 35 (1932).

The arrangement in which the above interferometers (1), (2) and (3) are conveniently and simultaneously adjusted is illustrated in Fig. 1. The incident monochromatic beam (5461A) is first divided by the half silvered mirror A. The two beams, os and op, are in turn divided by the half silvered mirrors B and B'. The full silvered auxiliary mirrors are D and D'. The quartz plate, whose front and back surfaces may be full silvered is represented by C. E, F and G are compensating glasses.

With A, B, B', C, D, and D' in proper adjustment, interference fringes from interferometer (1) are observed by opening shutters f_1 and f_2; those from interferometer (2), by opening f_1 and f_4; and those from interferometer (3) by opening f_2 and f_3. These shutters operate quickly and

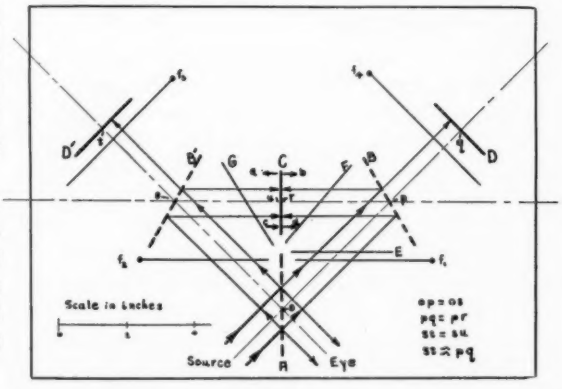

FIG. 1.

smoothly without affecting the tuning of the circuit for driving the quartz plate.

Flexural waves propagated in any direction in the plane of the front and back surfaces cause the plate to bend such that the direction of motion of opposing particles on the front and back surfaces is given by the arrows *c* and *d*. Standing longitudinal waves propagated in the plane of the plate give rise to alternate regions of compression and rarefaction which, on account of the tendency of the plate to maintain its volume, introduce motions of opposing particles on front and back surfaces as indicated by the arrows *a* and *b*. Longitudinal waves incident normally upon the front and back surfaces introduce the same type of motion, indicated by arrows *a* and *b*, of opposing particles on these surfaces. The direction of the arrows *a*, *b*, *c* and *d* is reversed at an instant one half period later.

Flexural vibrations therefore increase the length of one of the optical paths *su* and *rp* and decrease the other whereas longitudinal vibrations decrease or increase both of these paths simultaneously. Interferometer (1) consequently shows greater amplitudes for flexural vibrations than either interferometer (2) or (3). For symmetrical longitudinal modes, in which the amplitudes of vibration of opposing particles on the front and back surfaces are equal, interferometer (1) shows no fringe disturbance when the plate is set into vibration. For unsymmetrical longitudinal modes interferometer (1) shows in general much smaller amplitudes of vibration than either (2) or (3). The problem of distinguishing between the two modes of vibration is therefore reduced to the comparison of amplitudes of vibration. The latter has been thoroughly discussed in an earlier paper.[2]

It should be noted, however, that the primary function of the triple interferometer is to give direct information regarding the relative motion of those parts of the front and back surfaces whose images are superimposed in forming the interference fringes. The instrument has thus a broader purpose than merely to distinguish between the flexural and longitudinal modes.

The differentiation of flexual and longitudinal modes is illustrated by the photographs of Fig. 2 from crystals described in Table I.

[2] H. Osterberg, J.O.S.A. **22**, 35 (1932).

TABLE I.

Crystal	Cut	*x* in mm	*y* in mm	*z* in mm	Direction of electric field
I	X	4.32	22.46	25.24	X
II	X	4.32	22.47	25.77	X
14	X or Y	10.381	10.382	10.385	X

The crystal cube 14, which can be used as an X or Y-cut, was prepared by the writer for the double purpose of studying the vibrations of a cube and obtaining a simple longitudinal pattern. The orientation of the crystal faces with respect to the crystal axes is within 0.25 degree. Adjacent faces are perpendicular to the limits attainable with the more expensive squares. Opposite sides are parallel within a wave-length of sodium light.

The crystal faces perpendicular to the *X* or electrical axis were full silvered and connected across the variable condenser of a Hartley oscillator whose frequency was determined with a General Radio Precision Wavemeter, Serial No. 12, Type 224, recalibrated in August, 1932 by the Bureau of Standards.

In each of the following pairs of photographs, Fig. 2, the frequency and magnitude of the exciting potential are alike.

II *A* 1, II *A* 2, and II *A* 3 occur on crystal II at 361.6 kilocycles/sec. II *A* 1, from interferometer (1), clearly shows greater fringe disturbance than either II *A* 2 or II *A* 3 from interferometers (2) and (3) respectively. The pattern is flexural.

Patterns of the type II *A* 1 are common in quartz plates similar to I and II, in which the sides are not accurately perpendicular. These patterns are not the harmonics of a set of standing flexural waves propogated diagonally across the quartz plate. Using plates of excellent rectangular form, the writer has found that flexural waves are generally reflected at the crystal boundaries such that the angles of incidence and reflection are equal. The number and complexity of the patterns are markedly affected by the accuracy of the rectangular form of the plate.

I *A* 1, I *A* 2, I *A* 3 and I *A* 4 occur on crystal I; I *A* 1 and I *A* 2, at 247 kc per sec. from interferometers (1) and (2) respectively; I *A* 3 and I *A* 4, at 244 kc per sec. from interferometers (1) and

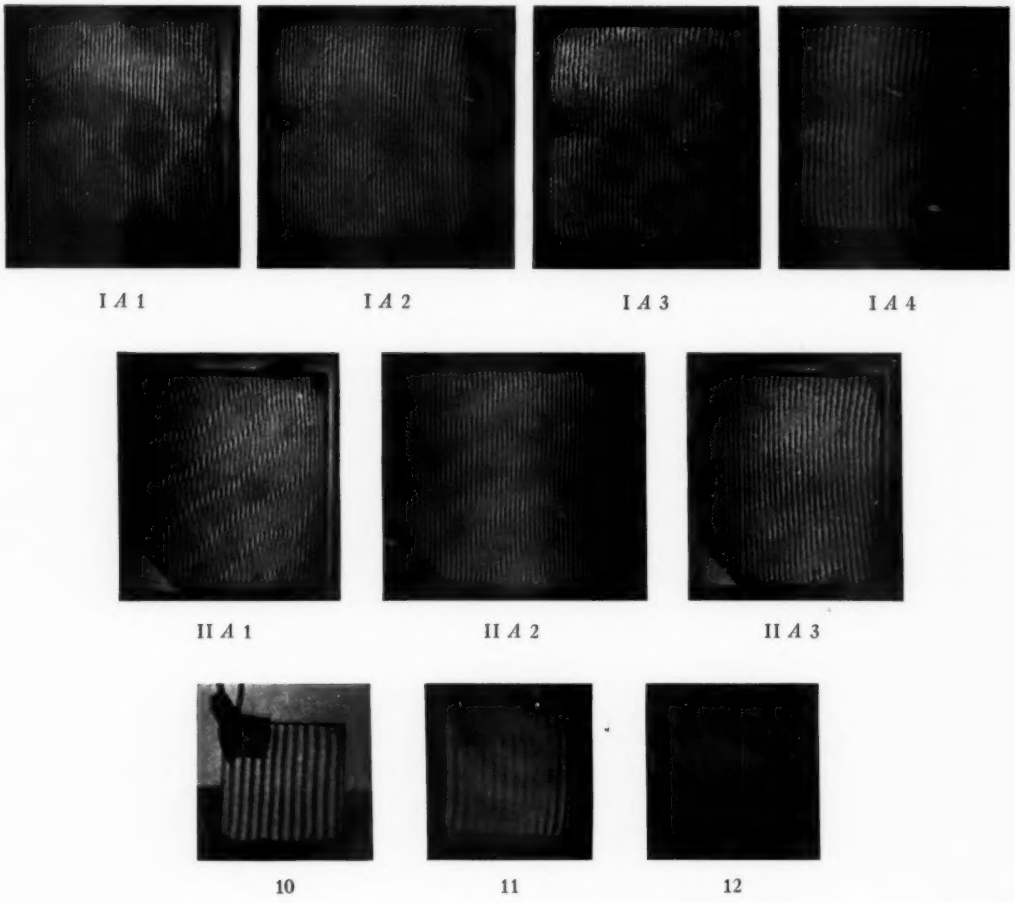

FIG. 2.

(2) respectively. Since both patterns are stronger in interferometer (1), they are flexural.

I *A* 1 and I *A* 3 form a beautiful demonstration of the coupling of mechanical modes in quartz plates. The third overtones of the sets of flexural waves propogated along the length and width of the crystal are here coupled to produce these patterns at 247 and 244 kc/sec. The uncoupled frequency of these overtones should fall within the interval 244–247 kc/sec.

In I *A* 1 that set of waves propagated along *y* is most sharply tuned; in I *A* 3, that set propagated along *z*, the optic axis.

Another characteristic common in such coupled modes is the shift of one-half wave-length experienced by one of the two coupled standing wave trains relative to its position in the crystal. Thus, the spots on I *A* 1 and I *A* 3 are seen to be shifted in position by one-half wave-length.

The pattern in 10, 11 and 12 occurs on crystal 14 at 210 kc/sec. 10 and 11 are from interferometers (1) and (2), respectively, and 12 from interferometer (2) at an amplitude of oscillation much greater than in 11. The fringe appearance in interferometer (1) is as in 10 for all amplitudes of oscillation. The pattern is therefore longitudinal. The front and back surfaces vibrate symmetrically but with an amplitude much greater at their central portions than toward the edges. The location of the antinodes on the *y* and

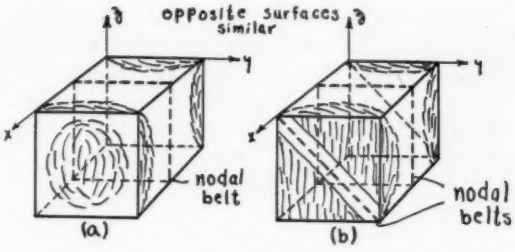

FIG. 3.

z faces, Fig. 3a, may support the assumption of a longitudinal pattern.

A serious difficulty arises in the previous pattern. The velocity of sound $v=n\lambda\equiv 2nx=4460$ meters/sec. in the plane perpendicular to the optic axis. This does not compare favorably with the value 5430 calculated from Young's modulus. Only the value 5430 is consistent with the mathematical solution of the vibrating bar given by P. Vigoureux[3] under the assumption that the influence of lateral expansions of the bar are negligible. That the lateral expansions in bars as short as crystal 14 are large, is obvious from the great variation of the amplitude of oscillation over the x faces and the motion, as well, of the y and z faces. Lateral expansion effects are more complicated when the frequency of these expansions approaches the frequency of a longitudinal (and perhaps, flexural) vibration propagated in the Y Z plane. This is known as the coupled cross mode and cannot be ignored in any adequate theory of the lateral effect. A coupling of the longitudinal fundamentals along X and Y is to be suspected in crystal 14 since $x=y$ and the Young's modulii E_x and E_y are equal.

The fundamental for the X direction was calculated from the relation $n=(1/2x)(E_x/\rho)^{\frac{1}{2}}=5430/2x=251$ kc/sec. A very active pattern, Fig. 3b, was found at this frequency. A rich variety of interesting patterns appears throughout the entire frequency spectrum of the cube. It is consequently hazardous to infer the character of a given pattern merely from the coincidence of the observed and computed frequencies.

[3] P. Vigoureux, *Quartz Resonators and Oscillators*, p. 40, 1931.

D. W. Dye[4] has found evidence that from 10 to 80 percent of the major surfaces remain at rest in the case of some quartz plates or bars oscillating, he supposes, at the fundamental longitudinal frequency. This is not consistent with the writer's experience unless these vibrations were very strongly coupled with cross modes.

The optical parts of Fig. 1 are mounted as in Fig. 4 in which the compensators G and F are omitted, B, B', and C are mounted on carriages which move on a single track T. The holders D and D' permit the variation of *pg* and *st*.

The simultaneous adjustment for good fringes in all three interferometers is simple provided: (a) no compensators are omitted, (b) the dividing mirrors A, B', and B are more than half silvered, and (c) the images from a pin hole are superimposed in the triple interferometer in any one of a number of systematic orders. The evaporation process[5] is recommended as the method of half silvering.

It is convenient but not necessary that A, B, B', D, D', E, and F be excellent optical flats. Thus in taking I A 1 and I A 3, D, D', G, and F were selected from $\frac{3}{8}$ in. plate glass.

The triple interferometer can be somewhat simplified. D' is not essential since interference from the left hand of C and D is easily adjusted. A feasible instrument and one more simply adjusted is one in which B and B' are full silvered but B, say, so hinged as to be swung smoothly and accurately in and out of the beam.

For measuring crystal patterns the writer has found the following interferometer simple, inexpensive, and very useful. A monochromatic light beam, Fig. 5a, is incident upon a glass plate B inclined at 45 degrees with the horizontal. The interference patterns are formed between the crystal C and the $\frac{1}{2}$ in. plate glass D and are measured from above with the aid of a long focus travelling microscope.

The various modes of oscillation are readily excited by the strip method in which the exciting potential is applied to light metal strips S, S, Fig. 5b. The crystal and strips are insulated upon sliders E, G, and F which are moved by screws K_1, K_2, and K_3.

[4] P. Vigoureux, *Quartz Resonators and Oscillators*, p. 57 1931.

[5] C. H. Cartwright and J. Strong, R. S. I. 2, 189 (1931).

One or two strips are usually employed both on the upper surface or one on each major surface. In the latter, the lower strip is held at Q between two pieces of plate glass mounted with Canada Balsam upon a third plate glass B. In exciting a given flexural harmonic, the position of the strips may or may not be critical, nor is the optimum position necessarily that in which the strip is located upon an antinode.

With this simple interferometer lycopodium powder can be strewn over the crystal while the interference pattern is observed. But dust was found to be a very uncertain means of exposing the various flexural overtones, becoming quite unusable at the higher overtones. Further, a large number of patterns too weak to cause appreciable movement of the dust give relatively strong interference patterns.

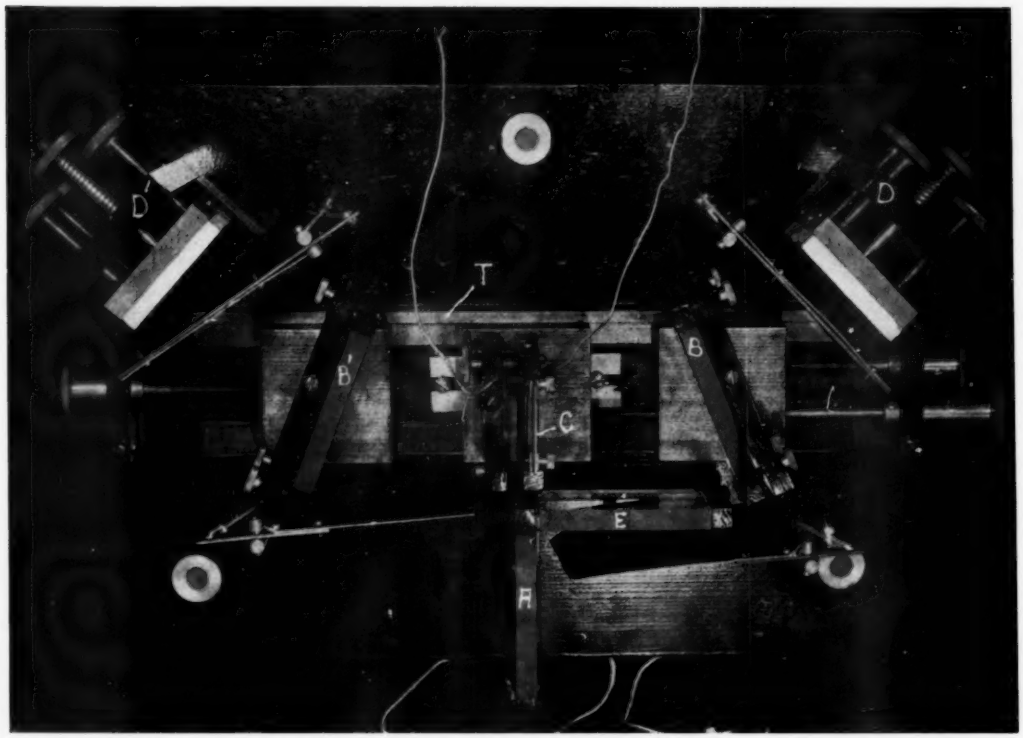

FIG. 4.

The application of the strip method of excitation to the triple interferometer presented the difficulty in that the crystal surfaces must be nonconducting. The destruction of fringe contrast in interferometer (1) resulting from the passage of light through an uncoated crystal, was avoided by evaporating a thin coat of copper upon the front and back surfaces and oxidizing this coat in an electric furnace. Copper oxide is sufficiently reflecting to produce good fringes in all interferometers provided D and D' are similarly coated. The complicated mechanism for moving the crystal and the exciting strips will be described elsewhere.

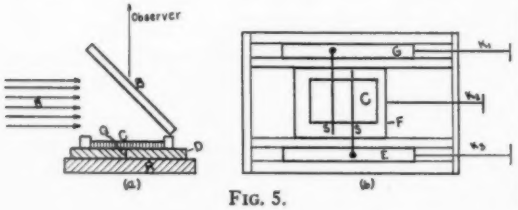

FIG. 5.